**ALGEBRA RH** **UNIT 15 REVIEW**

1. The square of a number decreased by 3 times the number is 28. Find all possible values of the number.

X = a number

$x^2-3x=28$
$x^2-3x-28=0$
$(x-7)(x+4)=0$
$x-7=0 \quad | \quad x+4=0$
$x=7 \quad | \quad x=-4$

The values of the number can be -4 or 7.

2. The sum of two numbers is 15 and difference of their squares is 45. Find both numbers.

x = one number = 9
(15-x) = other number = 6

$(x)^2-(15-x)^2=45$
$x^2-(225-30x+x^2)=45$
$x^2-225+30x-x^2=45$
$30x-225=45$
$30x=270$
$x=9$

3. Find two positive numbers whose ratio is 5:6 and whose product is 480

5x = one positive number = 20
6x = other positive number = 24

$5x(6x)=480$
$30x^2=480$
$x^2=16$
$\sqrt{x^2}=\pm\sqrt{16}$ not positive
$x=+4, \cancel{-4}$ reject

4. The product of two consecutive even integers is 48. Find all sets of integers that satisfy this description.

x = 1st consecutive even integer = -8, 6
(x+2) = 2nd consecutive even integer = -6, 8

$x(x+2)=48$
$x^2+2x=48$
$x^2+2x-48=0$
$(x+8)(x-6)=0$
$x+8=0 \quad | \quad x-6=0$
$x=-8 \quad | \quad x=6$

5. Find three consecutive positive integers such that the product of the first two is 22 less than 11 times the third.

x = 1st consecutive positive integer
(x+1) = 2nd consecutive positive integer
(x+2) = 3rd consecutive positive integer

$x(x+1)=11(x+2)-22$
$x^2+x=11x+22-22$
$x^2+x=11x$
$x^2-10x=0$
$x(x-10)=0$
$\cancel{x=0} \quad | \quad x-10=0$
reject $\quad | \quad x=10$

not positive

6. The perimeter of a rectangle is 32 cm and the area is 63 cm$^2$. Find the dimensions of this rectangle.

$\boxed{P = 32\text{ cm} \quad A = 63\text{ cm}^2}$

$P = 2L + 2W$

$32 = 2L + 2W$

$16 = L + W$

$x = \text{width} = 7, 9$

$16 - x = \text{length} = 9, 7$

$x(16 - x) = 63$

$16x - x^2 = 63$

$x^2 - 16x + 63 = 0$

$(x-9)(x-7) = 0$

$x - 9 = 0 \quad | \quad x - 7 = 0$

$x = 9 \quad | \quad x = 7$

Dimensions are 7 cm by 9 cm

7. A rectangular picture has a width that is two-thirds its length. The picture has an area of 294 square inches. What are the dimensions of the picture?

$\boxed{A = 294}$ $2x$, $3x$

$2x(3x) = 294$

$6x^2 = 294$

$x^2 = 49$

$\sqrt{x^2} = \pm\sqrt{49}$

$x = 7, \cancel{-7}$ reject

width $= 2x = 14$ in

length $= 3x = 21$ in

$\boxed{A = 294}$ $\frac{2}{3}x$, $x$

$\frac{2}{3}x(x) = 294$

$\frac{3}{2} \cdot \frac{2}{3}x^2 = 294 \cdot \frac{3}{2}$

$x^2 = 441$

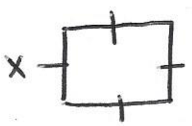
$\sqrt{x^2} = \pm\sqrt{441}$

$x = 21, \cancel{-21}$ reject

width $= \frac{2}{3}x = 14$ in

length $= x = 21$ in

8. A square is altered so that one dimension is increased by 4 and the other dimension is decreased by 2. The area of the resulting rectangle is 55. Find the area of the original square.




$x$ = side of the square

$x+4$

$\boxed{A = 55}$ $x-2$

one side of $\square = x + 4$

other side of $\square = x - 2$

$(x+4)(x-2) = 55$

$x^2 + 2x - 8 = 55$

$x^2 + 2x - 63 = 0$

$(x+9)(x-7) = 0$

reject $x + 9 = 0 \quad | \quad x - 7 = 0$

$\cancel{x = -9} \quad | \quad x = 7$

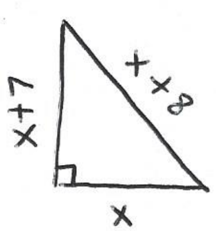
Area of square

$A = s^2$

$A = (7)^2$

$A = 49$ units$^2$

9. In a right triangle, the length of the longer leg is 7 more inches that the shorter leg. The length of the hypotenuse is 8 more inches than the length of the shorter leg. Find the perimeter of this right triangle.




$a^2 + b^2 = c^2$

$x^2 + (x+7)^2 = (x+8)^2$

$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$

$2x^2 + 14x + 49 = x^2 + 16x + 64$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x$ = shorter leg $= 5$

$x + 7$ = longer leg $= 12$

$x + 8$ = hypotenuse $= 13$

perimeter $= 30$ inches

$x = 5, \cancel{-3}$ reject

10. An object is moving such that it initially travels at a speed of 9 meters per second. It then speeds up at a rate of 2 meters per second each second. Under such conditions, the distance *d*, in meters, that the object travels is given by the equation $\boxed{d = t^2 + 9t}$ where *t* is in seconds. How long will it take the object to travel 22 meters?

$d$ = distance (meters)

$t$ = time (seconds)

$22 = t^2 + 9t$

$0 = t^2 + 9t - 22$

$0 = (t+11)(t-2)$

$t + 11 = 0 \quad | \quad t - 2 = 0$

$t = \cancel{-11} \quad | \quad t = 2$

reject

It takes the object 2 seconds to travel 22 meters

**11.** The profit $P$, in dollars, gained by selling $x$ computers is modeled by the equation $\boxed{P = -5x^2 + 1000x + 5000}$. How many computers must be sold to obtain a profit of $55,000?

P = profit (dollars)
x = # of computers

$55000 = -5x^2 + 1000x + 5000$
$0 = -5x^2 + 1000x - 50\,000$
$0 = -5(x^2 - 200x + 10\,000)$
$0 = -5(x-100)(x-100)$
$x - 100 = 0 \quad | \quad x - 100 = 0$
$x = 100 \quad | \quad x = 100$

To have a profit of \$55,000, 100 computers must be sold

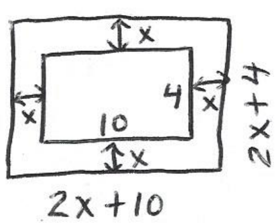
**12.** An object is launched straight up into the air at an initial velocity of 64 feet per second. Its height $H$, in feet, at $t$ seconds is given by the equation $\boxed{H = -16t(t-4) + 6.}$ Find all times $t$ that the object is at a height of 54 feet off the ground.

t = time (seconds)
h = height (feet)

$54 = -16t(t-4) + 6$
$54 = -16t^2 + 64t + 6$
$0 = -16t^2 + 64t - 48$
$0 = -16(t^2 - 4t + 3)$
$0 = -16(t-3)(t-1)$
$t - 3 = 0 \quad | \quad t - 1 = 0$
$t = 3 \quad | \quad t = 1$

The object is 54 feet off the ground at 1 second at at 3 seconds.

**13.** A wood floor is partially covered by a rectangular rug that is 4 ft by 10 ft. There is a uniform width of exposed flooring. If the total area of both the rug and exposed flooring is 112 square feet, find the dimensions of the floor.

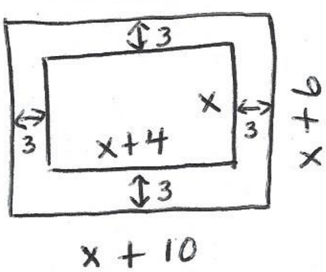



x = uniform width of flooring

dimensions of floor are $2x+10 = 2(2)+10 = 14$ ft and $2x+4 = 2(2)+4 = 8$ ft

$(2x+10)(2x+4) = 112$
$4x^2 + 8x + 20x + 40 = 112$
$4x^2 + 28x - 72 = 0$
$x^2 + 7x - 18 = 0$
$(x+9)(x-2) = 0$
reject $\cancel{x+9=0} \quad | \quad x - 2 = 0$
$\cancel{x=-9} \quad | \quad x = 2$

**14.** Andy wants to have a walkway installed around his rectangular pool. His pool is 4 feet longer than it is wide. The width of the walkway is going to be 3 feet. If the area of the pool is going to be the same as the area of the walkway, what are the dimensions of his pool? Round to the nearest tenth.




Area$_{\text{big rectangle}}$ − A$_{\text{pool}}$ = A$_{\text{walkway}}$
since the area of the walkway = area of pool

A big rectangle = 2 • A pool
$(x+10)(x+6) = 2(x)(x+4)$
$x^2 + 16x + 60 = 2x^2 + 8x$
$x^2 - 8x - 60 = 0$

$a = 1$
$b = -8$
$c = -60$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4(1)(-60)}}{2(1)}$

$x = \dfrac{8 \pm \sqrt{304}}{2} = \dfrac{8 \pm \sqrt{16}\sqrt{19}}{2} = \dfrac{8 \pm 4\sqrt{19}}{2} = 4+2\sqrt{19},\ 4-2\sqrt{19}$

$12.72,\ \cancel{-4.72}$ reject

width of pool = x = 12.7 ft
length of pool = x+4 = 16.7 ft

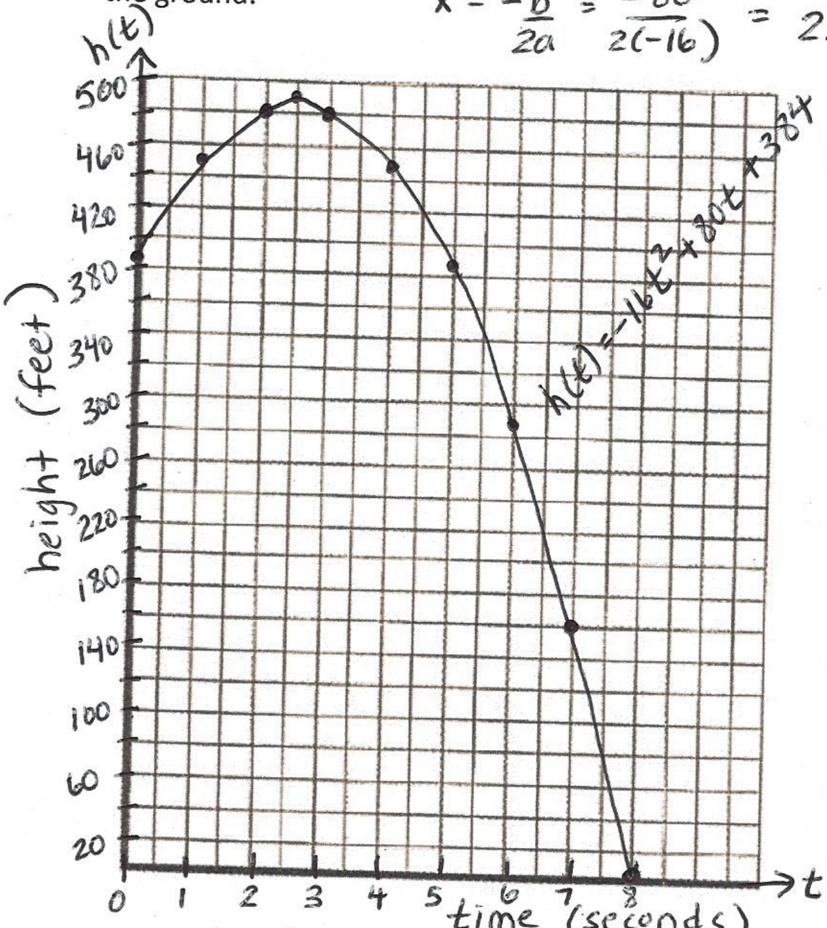
**15.** A rocket is launched from a cliff. The relationship between the height of the rocket $h(t)$, in feet, and the time since it is launched $t$, in seconds, can be represented by the function

$$h(t) = -16t^2 + 80t + 384.$$

$t$ = time (seconds)
$h$ = height (feet)

a) Using your calculator, sketch the graph of the rocket heights for all times where it is at or above the ground.

$x = \frac{-b}{2a} = \frac{-80}{2(-16)} = 2.5$ $h(2.5) = -16(2.5)^2 + 80(2.5) + 384$
$= 484$




| $t$ | $h(t)$ |
|---|---|
| 0 | 384 |
| 1 | 448 |
| 2 | 480 |
| 2.5 | 484 |
| 3 | 480 |
| 4 | 448 |
| 5 | 384 |
| 6 | 288 |
| 7 | 160 |
| 8 | 0 |

vertex

b) What is the maximum height of the rocket?

y value of vertex

484 feet

c) At what time will the rocket reach its maximum height?

x value of vertex

2.5 seconds

d) At what time will the rocket hit the ground?

root (zero)

8 seconds

e) For what time interval is the rocket increasing?

x values

(can't include starting or stopping points)

inequality notation $0 < x < 2.5$

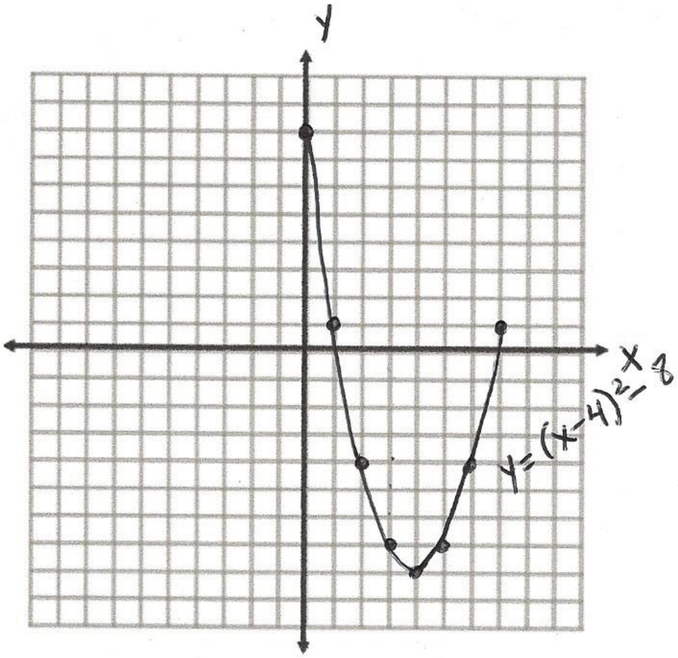
interval notation $(0, 2.5)$

**16.** A swimmer dives off a diving board in a path modeled by the quadratic function $\boxed{y = (x-4)^2 - 8}$ in the interval $\boxed{0 \le x \le 7.}$ The level of the surface of the water that she is diving into is represented by y = 0.

→ no arrows
restricted domain

vertex form
vertex: (4, −8)

a) Graph the quadratic for the given interval




| x | y |
|---|---|
| 0 | 8 |
| 1 | 1 |
| 2 | −4 |
| 3 | −7 |
| 4 | −8 |
| 5 | −7 |
| 6 | −4 |
| 7 | 1 |

b) How many vertical units below the surface of the water is the diver at the lowest point of the dive?

y value of minimum point (vertex)

8 units

c) What is the height of the diving board?

initial amount (y intercept)

8 units

d) To the nearest hundredth of a second, when did the diver reach the water's surface?

x value when y = 0

$0 = (x-4)^2 - 8$

$0 = x^2 - 8x + 16 - 8$

$0 = x^2 - 8x + 8$

$a = 1$
$b = -8$
$c = 8$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4(1)(8)}}{2(1)}$

$x = \dfrac{8 \pm \sqrt{32}}{2}$

$x = \dfrac{8 \pm 4\sqrt{2}}{2} = 4 \pm 2\sqrt{2}$

$4 + 2\sqrt{2}$
6.828
6.3 seconds
(coming up)

$4 - 2\sqrt{2}$
1.171
1.17 seconds
(going in)

**17.** A ball is thrown into the air with an initial upward velocity of 48 $ft/s$. Its height h in feet after $t$ seconds is given by the function $\boxed{h(t) = -16t^2 + 48t + 4}$

$t$ = time (seconds)
$h$ = height (feet)

a. From what height was the ball thrown?

y intercept 4 feet

b. What does the coefficient of $t^2$ tell you about the end behavior of the ball?

eventually, the graph will turn down (law of gravity)
the end behavior approaches $-\infty$

c. What is the height of the ball after 2 seconds have passed?

$t=2$

$h(2) = -16(2)^2 + 48(2) + 4$
$= 36$ feet

d. What is a reasonable domain and range for this situation? (nearest hundredth)

domain: Find the zero(s) of the graph

$y = -16t^2 + 48t + 4$
$0 = -16t^2 + 48t + 4$
$0 = -4(4t^2 - 12t - 1)$

$a = 4$
$b = -12$
$c = -1$

$x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(4)(-1)}}{2(4)}$

$x = \dfrac{12 \pm \sqrt{160}}{8}$

$x = \dfrac{\cancel{12}^{3} \pm \cancel{4}^{1}\sqrt{10}}{\cancel{8}_{2}}$

$x = \dfrac{3+\sqrt{10}}{2}, \dfrac{3-\sqrt{10}}{2}$

$\approx 3.08$ $\quad -0.08$ reject

$0 \le x \le 3.08$

$[0, 3.08]$

range: Find the vertex of the graph

$x = \dfrac{-b}{2a} = \dfrac{-48}{2(-16)} = 1.5$

$y = -16(1.5)^2 + 48(1.5) + 4$
$= 40$

vertex $(1.5, 40)$ highest height

$0 \le y \le 40$

$[0, 40]$