1)

a) Graph $y = -x^2 - 5x + 3$ using a table of values.

b) State and graph the equation of the **axis of symmetry**. $x = -2.5$

$$x = \frac{-b}{2a}$$
$$x = \frac{-(-5)}{2(-1)}$$
$$x = -2.5$$

c) Determine the coordinates of the **turning point**. **(-2.5, 9.25)**

Substitute the value of x into the equation to find the value of y.
$y = -(-2.5)^2 - 5(-2.5) + 3$
$y = 9.25$

d) State whether the vertex is a *maximum* or a *minimum* point. **maximum**

State the **roots** of the parabola (round to the nearest hundredth). **{≈-5.54, ≈0.54}**

In this situation, the roots cannot be identified from the graph. The quadratic equation cannot be factored. To find the roots, use the quadratic formula.

$0 = -x^2 - 5x + 3$ a = -1, b = -5, c = 3

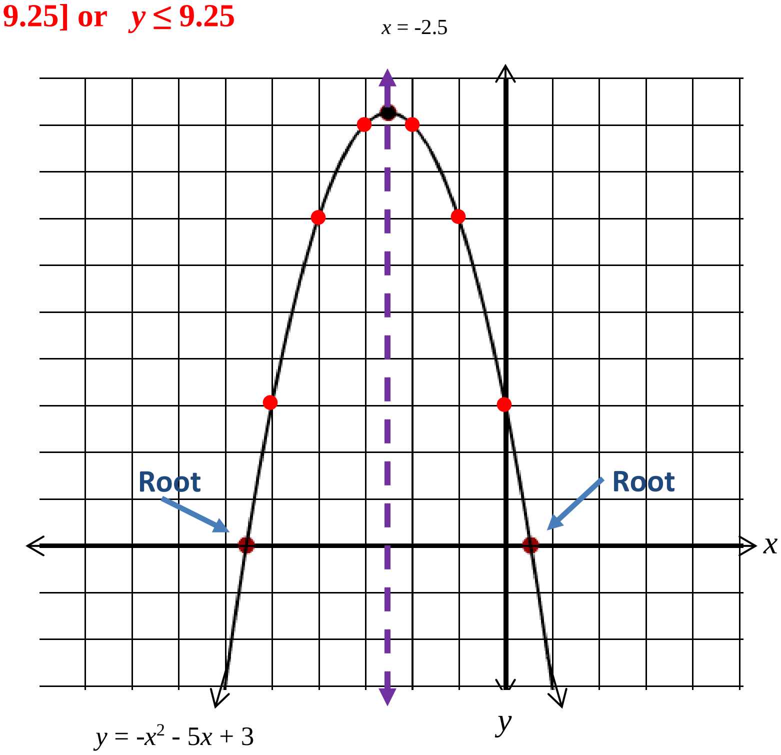
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-1)(3)}}{2(-1)}$$

$$x = \frac{5 \pm \sqrt{37}}{-2}$$
$$x = \frac{5 + \sqrt{37}}{-2} \approx -5.54$$
$$x = \frac{5 - \sqrt{37}}{-2} \approx 0.54$$

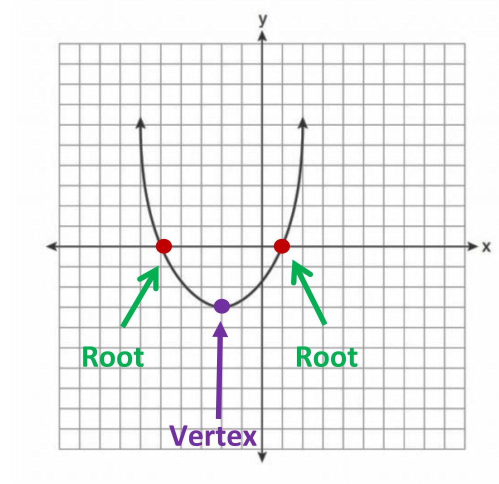
e) State the **y-intercept** of the graph. **(0, 3)**

f) State the **range** of the function. **(-∞, 9.25] or $y \leq 9.25$**

x	y
-6	-3
-5	3
-4	7
-3	9
-2.5	9.25
-2	9
-1	7
0	3
1	-3



2) Let ***f*** be the function represented by the graph.



a) State the **roots** of the function. **{-5, 1}**

b) State the **vertex**. **(-2, -3)**

c) Let ***g*** be a function such that $\mathbf{g(x) = \frac{1}{2}x^2 + 4x + 3}$.

Determine which function has the smaller *minimum* value. Justify your response.

$$x = \frac{-b}{2a}$$
$$x = \frac{-(4)}{2(0.5)}$$
$$x = \frac{-4}{1}$$
$$x = -4$$

$g(-4) = 0.5(-4)^2 + 4(-4) + 3$

$g(-4) = -5$ **(-4, -5)**

***g*(*x*) has the smaller minimum value since -5 < -3.**

3) Each quadratic function below has a *domain* of all real numbers. State the ***range*** of each function.

h(x) = – (x + 2)^2 + 5

vertex: (-2, 5)
maximum
range: ***h*(*x*) ≤ 5**

(-∞, 5]

x	*y*
-3	1
-2	-5
-1	**-7**
0	-5
1	1

vertex: (-1, -7)
minimum
range: **y ≥ -7**

[-7, ∞)

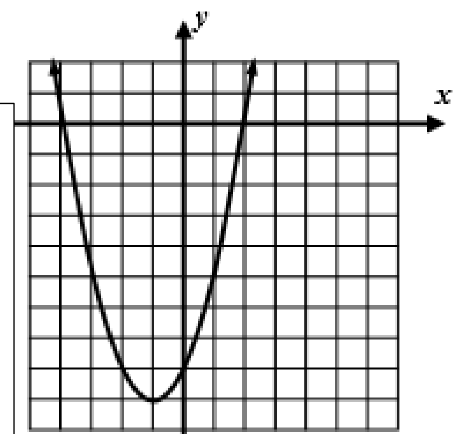
f(x) = 4x^2 + 8x – 6

vertex: (-1, -10)
minimum
range: ***f*(*x*) ≥ -10**

[-10, ∞)

Find the coordinates of the vertex using

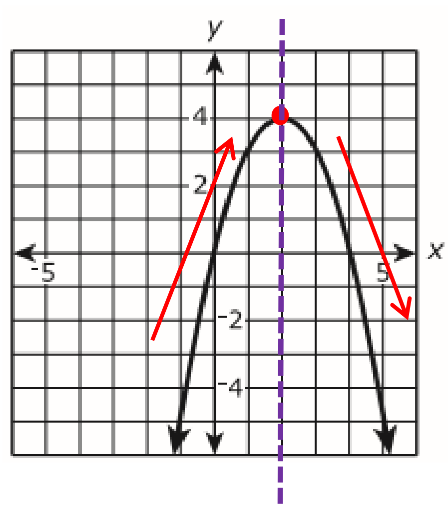
$$x = \frac{-b}{2a}$$



vertex: (-1, -9)
minimum
range: **y ≥ -9**

[-9, ∞)

4) The graph of the function $\mathbf{f(x) = 4x - x^2}$ is shown here.



a) State the ***range*** of the function. ***f*(*x*) ≤ 4 or (-∞, 4]**

b) State the interval on which *f*(x) is ***increasing***.

***x* < 2**

c) State the interval on which *f*(x) is ***decreasing***.

***x* > 2**

5) State the **zeros** of the function ***f*(x) = – 10(x + 3)(x – 7)**. {-3, 7}

6) In the xy-coordinate plane, the graph of the equation **y = 3x² – 12x – c** has zeros at x = a and x = b, where a < b. The graph has a minimum at (2, -48). What are the values of a, b and c?
vertex

1st: *Substitute (2, -48) for x and y and solve for c.*
2nd: *Find the roots (zeros) of the function to find a and b.*

$$y = 3x^2 - 12x - c$$
$$-48 = 3(2)^2 - 12(2) - c$$
$$-48 = 12 - 24 - c$$
$$-48 = -12 - c$$
$$-36 = -c$$
$$c = 36$$

*Replace **c** with 36 and find the roots.*

$$y = 3x^2 - 12x - 36$$
$$0 = 3x^2 - 12x - 36$$
$$0 = 3(x^2 - 4x - 12)$$
$$0 = 3(x - 6)(x + 2)$$

$x - 6 = 0$	$x + 2 = 0$
$x = 6$	$x = -2$

***a* = -2**

***b* = 6**

***c* = 36**

7) Given the function ***f*(x) = x² + 6x + 8**

a) Rewrite the function in **factored form**. State the **zeros** of the function.

$f(x) = (x + 2)(x + 4)$ **{-2, -4}**

b) Rewrite the function in **vertex form** by completing the square. State the **vertex** of the function.

$$f(x) = x^2 + 6x + 8$$
$$y - 8 + \_\_\_ = x^2 + 6x + \_\_\_$$
$$(x + 3)(x + 3)$$
$$y - 8 + 9 = x^2 + 6x + 9$$
$$y + 1 = x^2 + 6x + 9$$
$$y + 1 = (x + 3)^2$$
$$y = (x + 3)^2 - 1$$
$$f(x) = (x + 3)^2 - 1$$

vertex: (-3, -1)

c) Rewrite the function **y = 2x² – 8x + 6** in **vertex form** by completing the square. State the **vertex** of the function.

$$y = 2x^2 - 8x + 6$$

$$\frac{y}{2} = \frac{2x^2}{2} - \frac{8x}{2} + \frac{6}{2}$$

$$\frac{y}{2} = x^2 - 4x + 3$$

$$\frac{y}{2} - 3 + \_\_\_ = x^2 - 4x + \_\_\_$$

$$(x-2)(x-2)$$

$$\frac{y}{2} - 3 + 4 = x^2 - 4x + 4$$

$$\frac{y}{2} + 1 = x^2 - 4x + 4$$

$$\frac{y}{2} + 1 = (x-2)^2$$

$$\frac{y}{2} = (x-2)^2 - 1$$

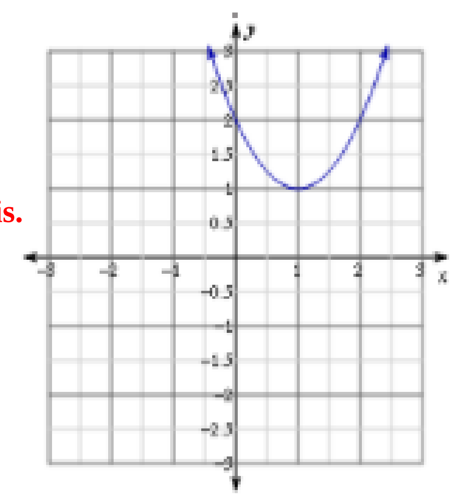
$$\frac{2}{1} \bullet \frac{y}{2} = 2\left[(x-2)^2 - 1\right]$$

$$y = 2(x-2)^2 - 2$$

d) Which function has the smaller *minimum* value?

$y = 2x^2 - 8x + 6$ has the smaller minimum value because -2 < -1.

8) Answer a and b based on the graph shown below.



a) Are the roots *real* or *non-real* numbers?

Non-real roots because the parabola does not intersect the *x*-axis.

b) Is the discriminant ($b^2 - 4ac$) *positive* or *negative*? **negative**

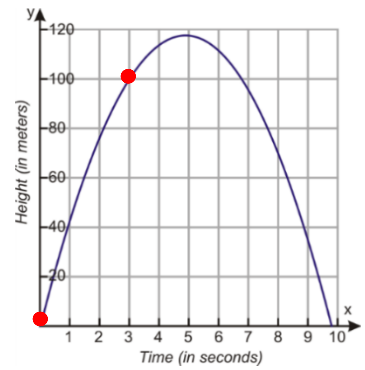
9) The height of an object after it has been launched is modeled by the graph of the quadratic function shown here where **y** represents the height of the object from the ground after **x** seconds.

Calculate the *average rate of change* of the height of the object for the first 3 seconds after being launched.

(0, 0) (3, 100)

$$\frac{\Delta y}{\Delta x} = \frac{100-0}{3-0} \longrightarrow 33.\bar{3}\text{ mps}$$

average rate of change: $33.\bar{3}$ mps



10) A student throws a bag of chips to her friend. Unfortunately, her friend does not catch the chips, and the bag hits the ground. The distance from the ground (height) for the bag of chips is modeled by the function $h(t) = -16t^2 + 32t + 4$, where ***h* is the height** (distance from the ground in feet) of the chips, and ***t* is the number of seconds** the chips are in the air.

a) From what height are the chips being thrown? *y-intercept* (0, 4) → **4 feet**

b) What is the maximum height the bag of chips reaches while airborne? *vertex* (1, 20)→ **20 feet**

$$t = \frac{-b}{2a} = \frac{-(32)}{2(-16)} = \frac{-32}{-32} = 1$$

$$h(1) = -16(1)^2 + 32(1) + 4$$
$$h(1) = 20$$

c) How long does it take the bag of chips to reach its maximum height? *vertex* (1, 20)→ **1 second**

All parts of this question can also be answered by viewing the table of values for the function.

x (time, seconds)	*y (height, ft)*
0	**4**
1	**20**
2	4

The chips are thrown from a height of 4 feet.
The chips reach their maximum height after 1 second.

11) A rocket is launched from a cliff. The relationship between the height of the rocket, *in feet*, and the time since its launch, ***t***, *in seconds* can be represented by the function $\boldsymbol{h(t) = -16t^2 + 80t + 384}$. How long did it take for the rocket to hit the ground?

zeros { -3, 8 } → reject -3, **8 seconds**

$$h(t) = -16t^2 + 80t + 384$$
$$0 = -16t^2 + 80t + 384$$
$$0 = -16(t^2 - 5t - 24)$$
$$0 = -16(t-8)(t+3)$$

$t - 8 = 0$	$t + 3 = 0$
$t = 8$	$t = -3$
	reject

This question can also be answered by viewing the table of values.

***x* (time, seconds)**	***y* (height, ft)**
0	384
1	448
2	480
3	480
4	448
5	384
6	288
7	160
8	**0**

*At **8 seconds**, the height of the rocket is **0 feet** which means it has hit the ground.*