Unit 15

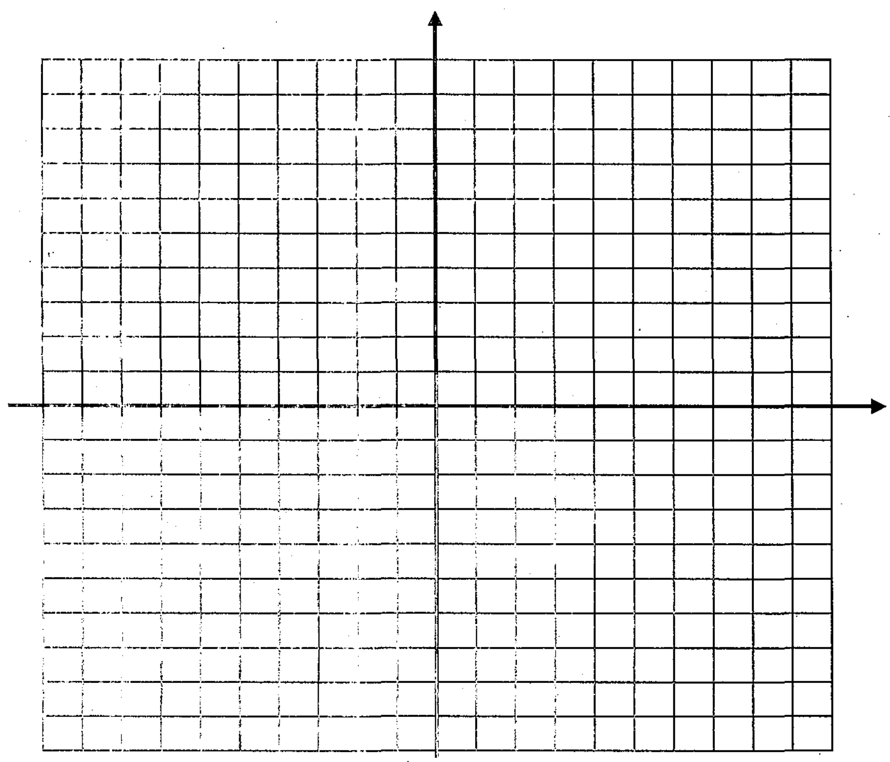
Graphing Quadratic Functions

- Make sure your equation is written in **standard form** ($y = ax^2 + bx + c$).
- Find the **x-coordinate** of the **vertex** (turning point) using the formula, $x = \frac{-b}{2a}$.
- Create a **table of values** using three x-values less than the x-value of the vertex, and three x-values greater than the vertex.
- **Graph** the points from the table of values and connect them with a **smooth curve**.
- **Label** the parabola with the equation.

1) Graph the quadratic function $y = x^2 - 6x + 4$

- Find the coordinates of the vertex.

- Create a table of values.



IMPORTANT NOTE

Every parabola is **symmetrical**. If a *vertical line* were drawn through the vertex, it would divide the parabola into <u>two equal halves</u>.

This *vertical line* is known as the **axis of symmetry**.

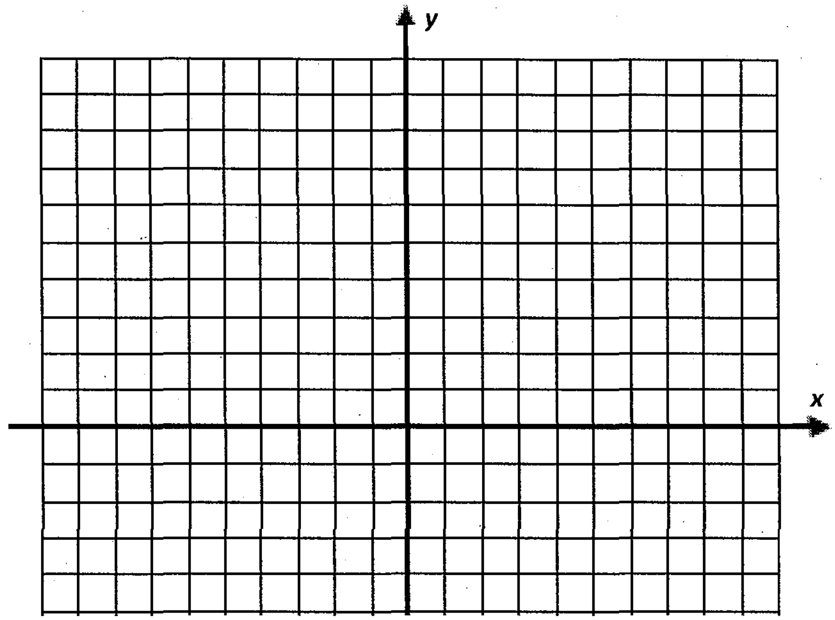
Graph the axis of symmetry in the above example and state the equation of the line.

Essential Question: What does the graph of a quadratic function look like?

Do Now: Graph the following quadratic function using the domain $-3 \le x \le 3$ and answer the questions that follow.

$$y = x^2$$

x	y
-3	
-2	
-1	
0	
1	
2	
3	

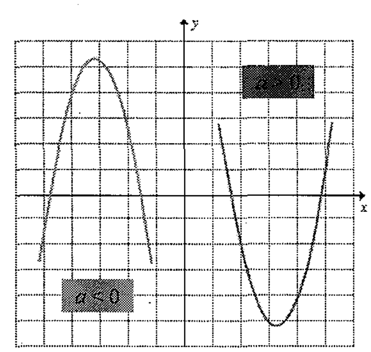


1) Is the function linear?

2) Is the function increasing or decreasing?

3) Does the function have a constant rate of change?

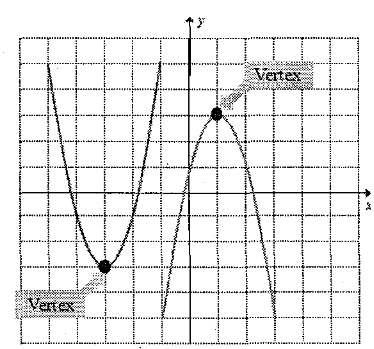
Quadratic Functions



- The **standard form** of a quadratic function is ____________________.
- The graph of a quadratic function is a ____________________.
- The **parabola** can open __________ or __________.
- It opens **up** when the ***a*** value is ____________________ and opens **down** when the ***a*** value is ____________________.
- The turning point of a parabola is known as the ____________________.

  When parabolas open up, the **vertex** is the *minimum* point.

  When parabolas open down, the **vertex** is the *maximum* point.



- The **x-value** of the turning point (x, y) can be calculated using the formula:

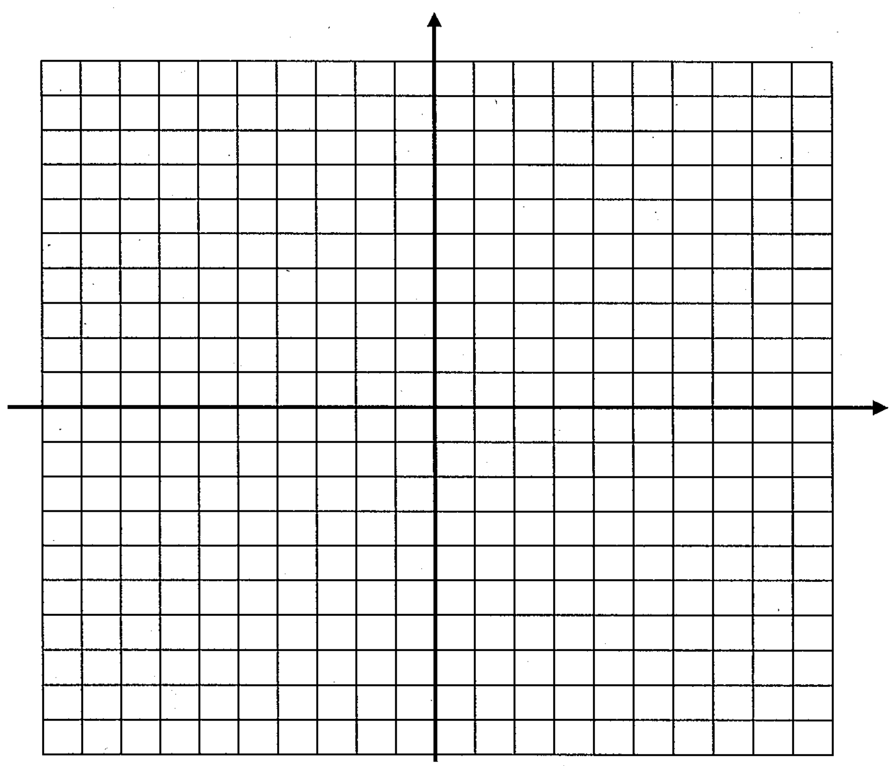
Graphing Quadratic Functions

- Make sure your equation is written in **standard form** ($y = ax^2 + bx + c$).
- Find the **x-coordinate** of the **vertex** (turning point) using the formula, $x = \frac{-b}{2a}$.
- Create a **table of values** using three x-values less than the x-value of the vertex, and three x-values greater than the vertex.
- **Graph** the points from the table of values and connect them with a **smooth curve**.
- **Label** the parabola with the equation.

1) Graph the quadratic function $y = x^2 - 6x + 4$

- Find the coordinates of the vertex.

- Create a table of values.



IMPORTANT NOTE

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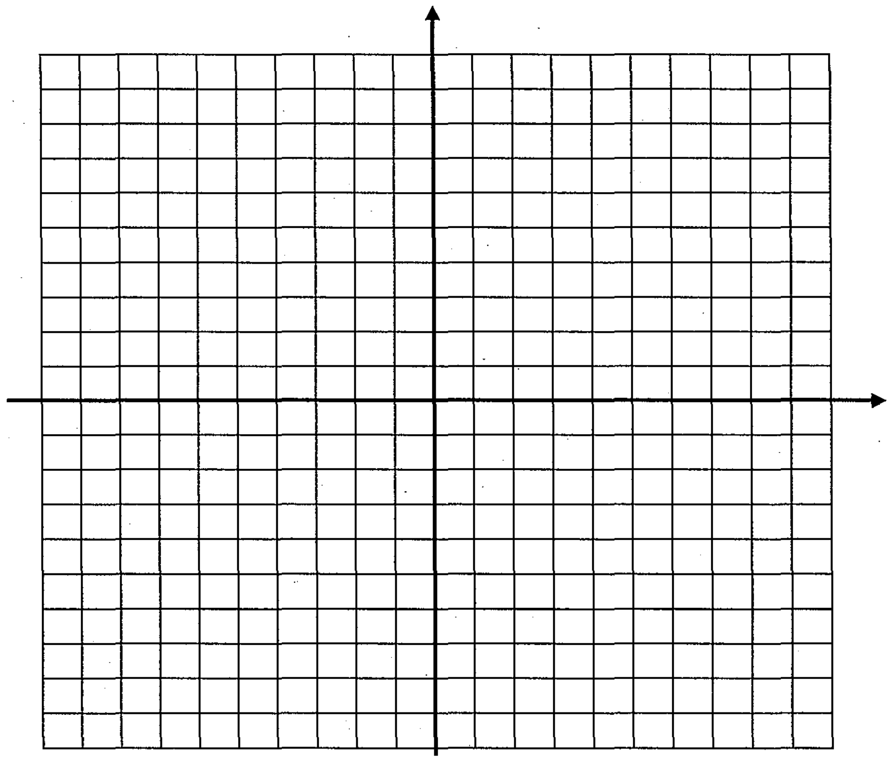
This *vertical line* is known as the **axis of symmetry**.

Graph the axis of symmetry in the above example and state the equation of the line.

2) Graph the function $f(x) = -2x^2 + 3$

- Find the coordinates of the vertex.

- Create a table of values.



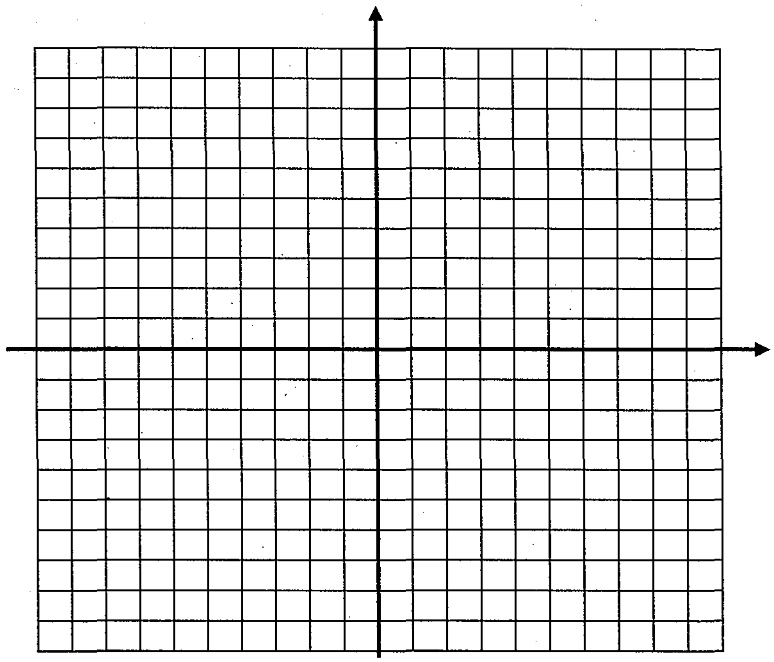
- Graph the **axis of symmetry** and state the equation of the line.

- The value of _________ in $y = ax^2 + bx + c$ causes the quadratic to open upward or downward.
- A positive value of ________ tells us that the graph will open ______________.
- A negative value of ________ tells us that the graph will open ______________.
- The turning point of a parabola is known as the ______________________________.
- This vertex is either the _______________________ point or ________________________ point of the graph depending on whether the parabola opens up or down.
- The x-coordinate of the vertex can be found using the formula ________________.
- The ____ is a vertical line that divides the parabola into two equal halves.

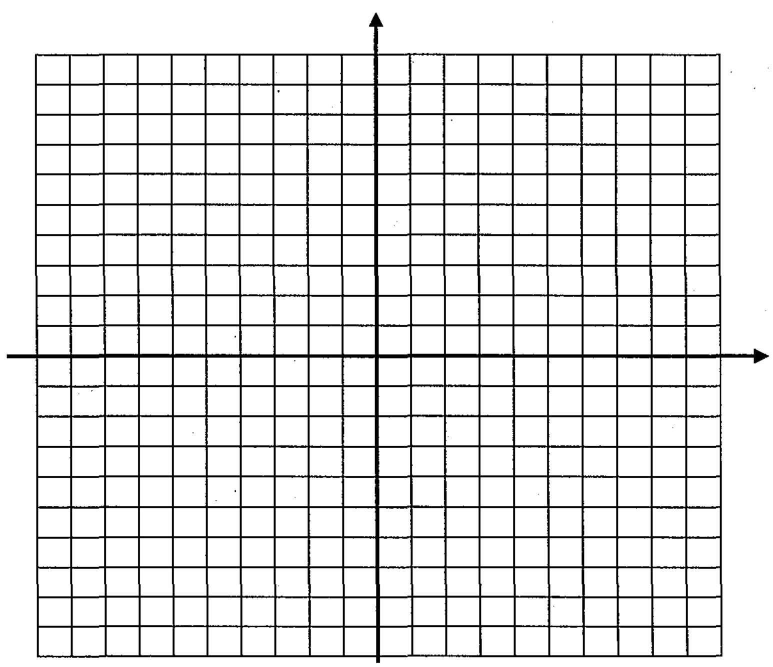
For the following quadratic functions:

- Create a table of values
- Graph the parabola
- Label the vertex and determine if it is a *minimum* or *maximum* point
- Graph and label the axis of symmetry

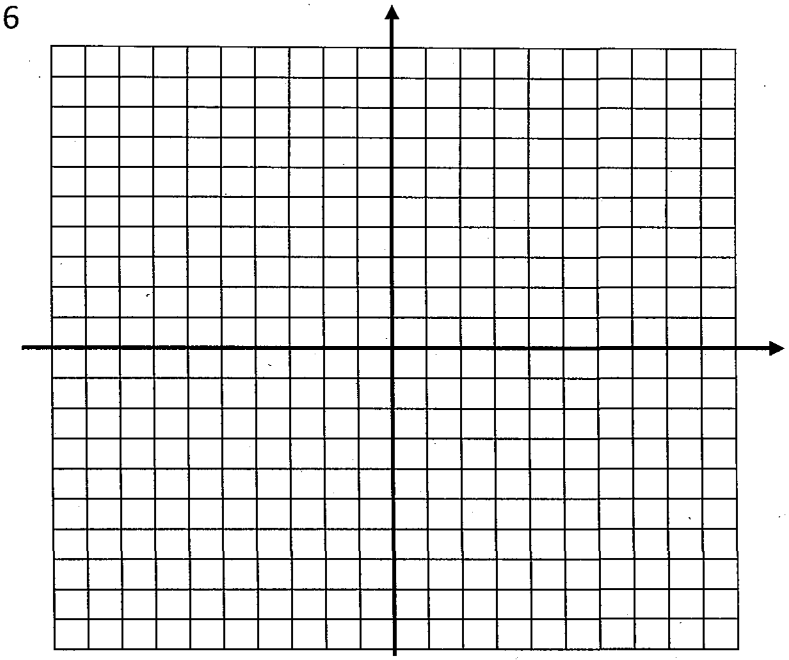
1) $y = x^2 - 2x - 8$



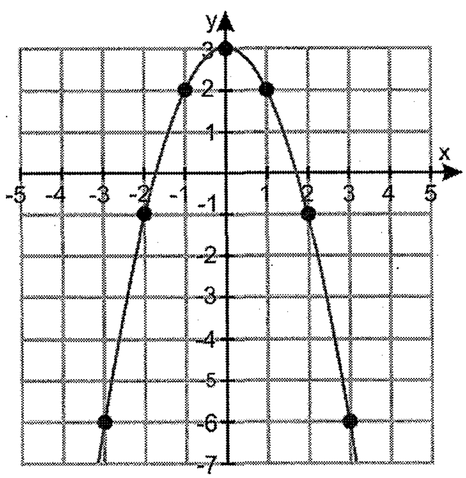
2) $y = -x^2 + 4$



3) $g(x) = \frac{1}{2}x^2 - 4x + 6$



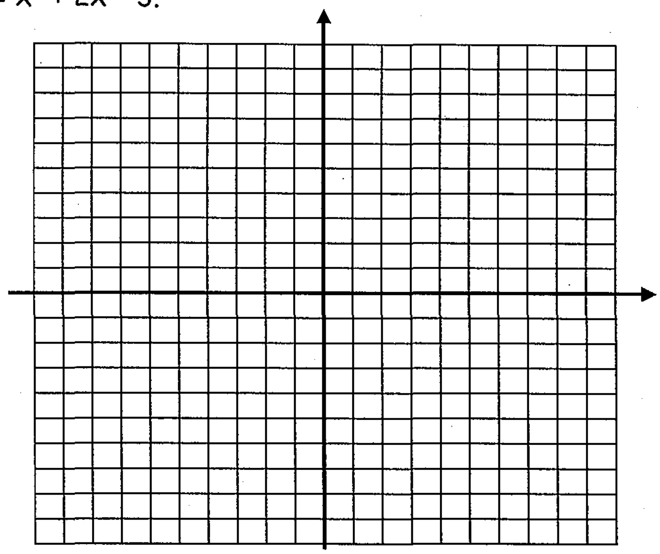
4) Which function has a larger *maximum*?
Justify your response.



$f(x) = -x^2 + 2x + 1$

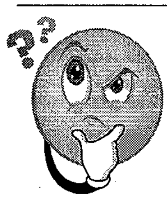
Essential Question: What are the roots of a quadratic function?

Do Now: Graph the function $y = x^2 + 2x - 3$.



Where does the graph intercept the *x*-axis?

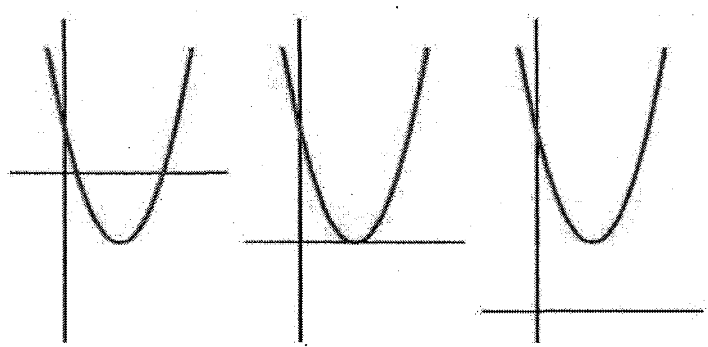
x-intercepts: (______, 0) (______, 0)



Think about this…. What is the solution set to the equation $x^2 + 2x - 3 = 0$?

What do you notice about the **solution set** to the equation and the **x-intercepts** of the graph from the Do Now?

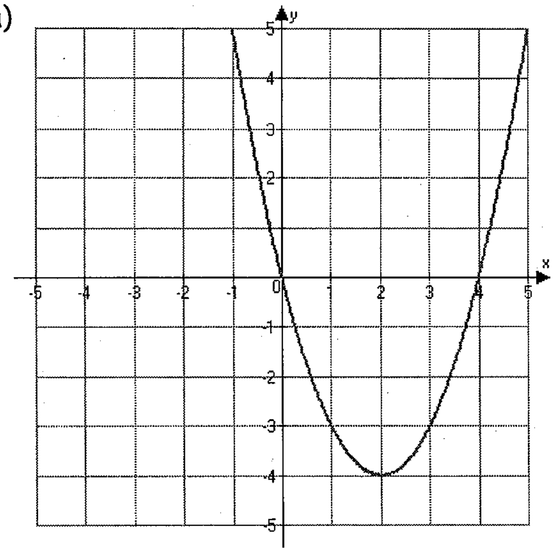
The "**roots**" of a parabola are the ***x*-coordinates** of the points where the curve intercepts the x-axis. These values are also known as the "**zeros**" of the function.



- If a parabola intersects the x-axis twice, then the parabola has ______ real roots.
- If it only intersects the x-axis once, then the parabola has ______ real root.
- If the parabola doesn't intersect the x-axis, then the roots are _______ ____________.

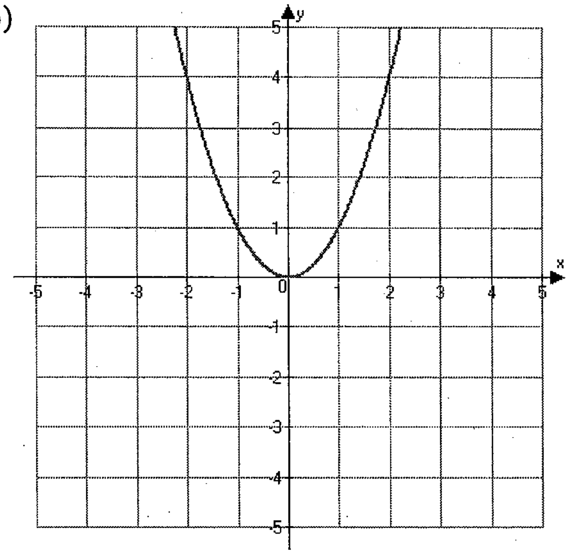
1) Identify the **roots** of the quadratic functions graphed below?

a)



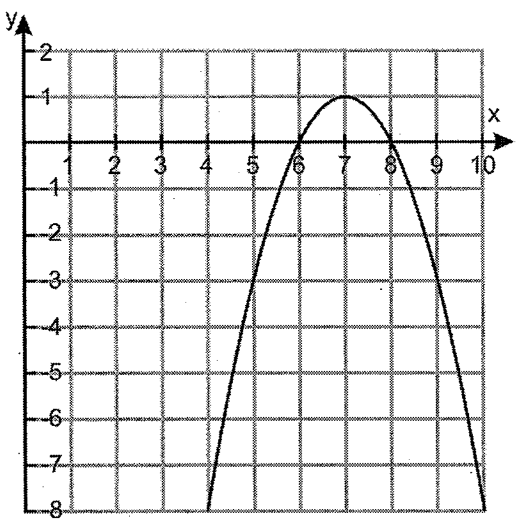
Roots:__________________________

b)



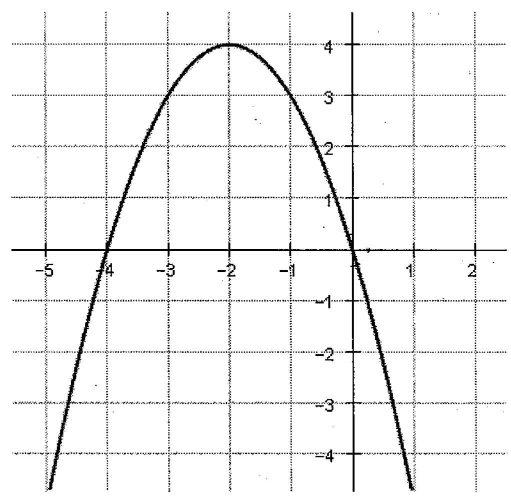
Roots:__________________________

c)



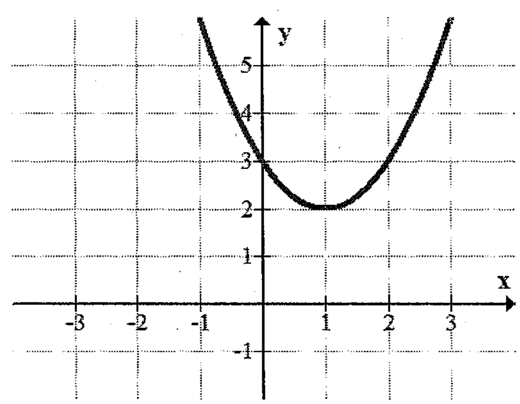
Roots:__________________________

d)



Roots:__________________________

2) Jeremy says that the parabola shown here has one root, $x = 3$.
Do you agree or disagree? Explain.



3) Without graphing, how can we find the roots of the quadratic function $y = 3x^2 + 18x + 24$?

4) Find the roots of the function $y = x^2 + 3x - 18$ in two ways.

Algebraically

Graphically (*use calculator*)

2^{nd} Trace (Calc)
Option 2: Zero
Move cursor left of the x-intercept (left bound)
Enter
Move cursor right of the x-intercept (right bound)
Enter
(Guess) Enter

Table of Values
Look for (x, 0)

The **roots** of a quadratic function can be found ______________________________

by identifying the **x-value** of the **x-intercepts** and ______________________________

by replacing **y** in $\mathbf{y = ax^2 + bx + c}$ with ________ and solving for x.

JUST ONE more THING!

The **discriminant** is a numerical value that provides information about the roots of a quadratic function. It is calculated using the formula $\mathbf{b^2 - 4ac}$.

- If $\mathbf{b^2 - 4ac}$ is **positive**, then the **roots are real** numbers.
- If $\mathbf{b^2 - 4ac}$ is **negative**, then the **roots are *not real*** (*the graph does not intersect the x-axis*).

Examples: Determine if the roots of the functions are real or not real.

$\mathbf{y = x^2 - 2x + 5}$
$b^2 - 4ac$
$(-2)^2 - 4(1)(5)$
-16 ←This value tells me that the graph will *not* intersect the x-axis (*no real roots*)

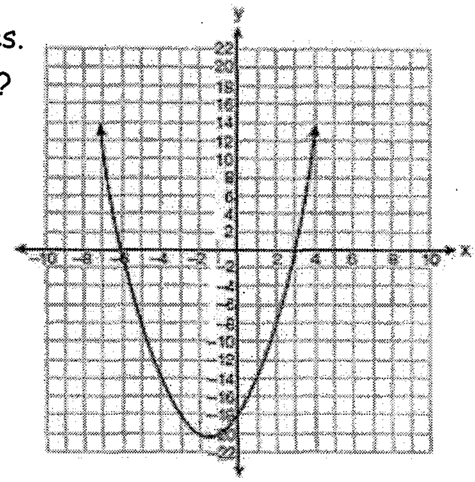
$\mathbf{y = x^2 + 9x + 14}$
$b^2 - 4ac$
$(9)^2 - 4(1)(14)$
25←This value tells me that the graph will intersect the x-axis (real roots)

See graphs on calculator.

1. The function $y = x^2 + 3x - 18$ is graphed on the set of axes. Based on the graph, what are the roots of this parabola?

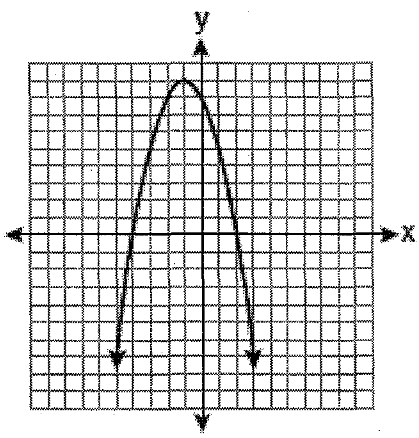
   (1) –3 and 6 (2) 0 and –18

   (3) 3 and –6 (4) 3 and –18

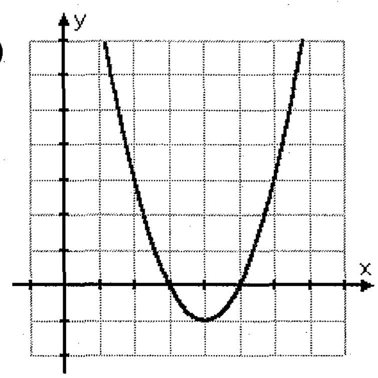


2. What are the root(s) of the parabolas graphed below?

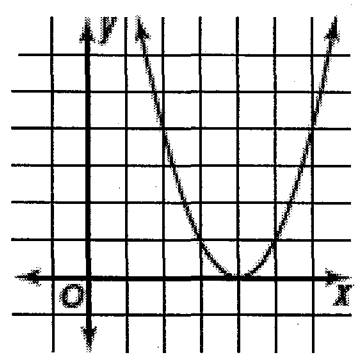
a)



b)



c)



3. Determine the roots of the function **f(x) = -x² + 8x + 9** *algebraically*. Check your answer with a graph or table of values.

4. How many *real roots* does the quadratic function **h(x) = -x² + 2x – 3** have? Justify your response.

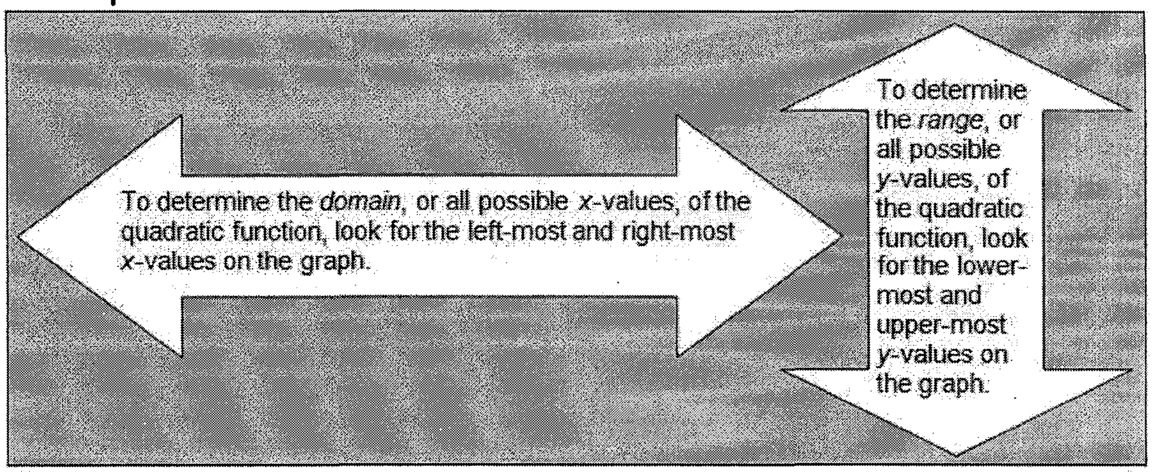
Essential Questions: How do we determine the domain and range of a quadratic function? How do we determine when the function is increasing? How do we determine when the function is decreasing?

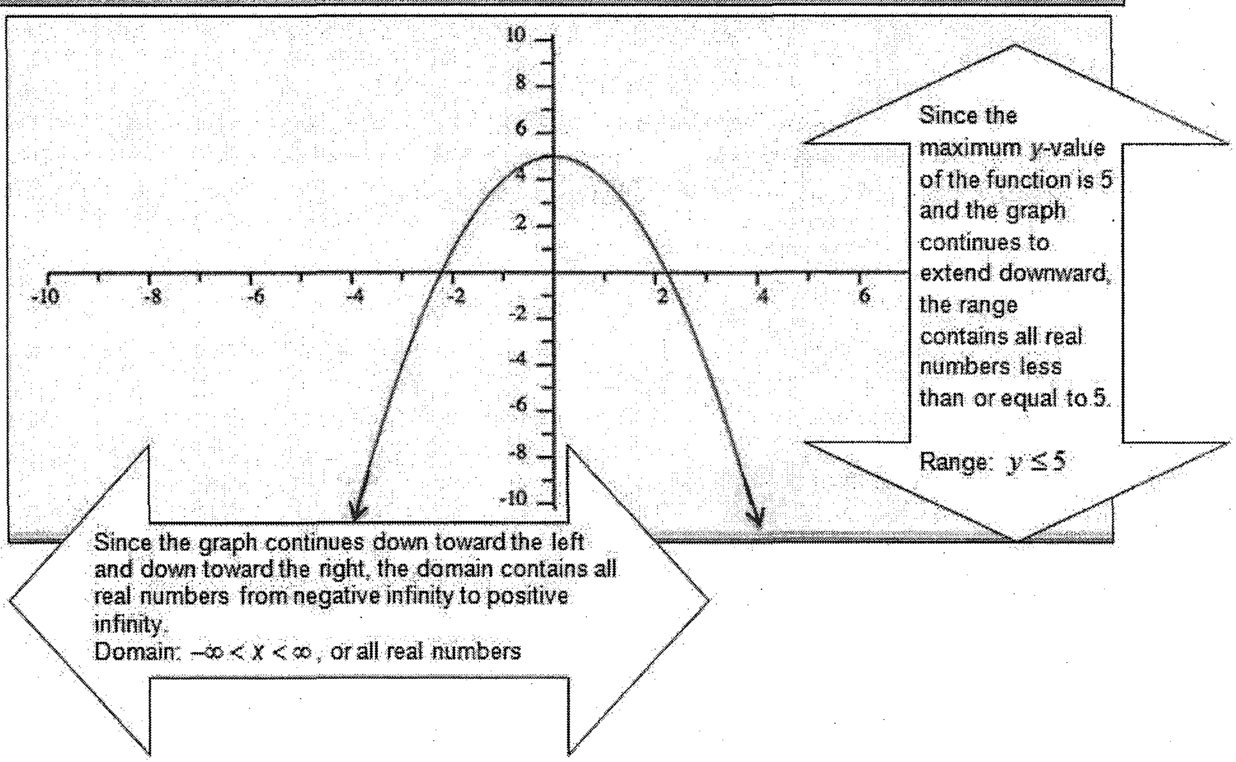
Do Now: Complete the table with the correct terminology.

Domain		x-values
	output	

Domain and Range of Quadratic Functions

Substituting any real value of x into a quadratic equation results in a real number. Therefore, in general, the domain of any quadratic function is *all* real numbers. The range of a quadratic function depends on its vertex and the direction that the parabola opens.

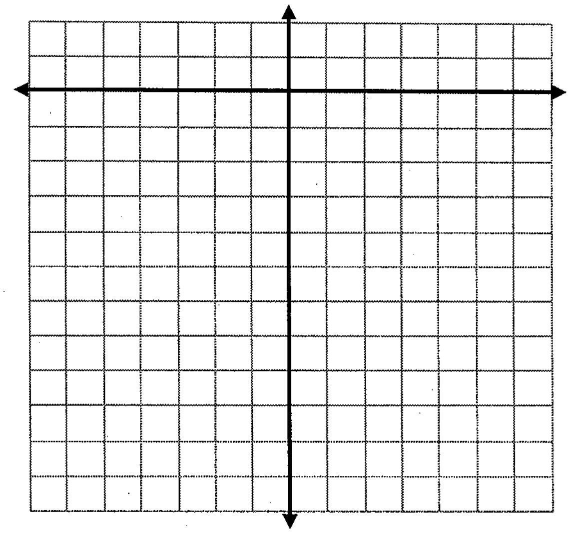




1) Graph the quadratic function $y = -x^2 + 4x - 6$

State the:

- Vertex: _______________
- Maximum or minimum_____________
- x-intercepts: __________________
- Zeros (roots): ____________
- Domain: _________________
- Range: __________________

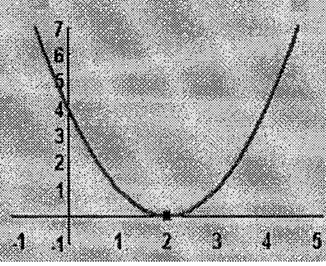


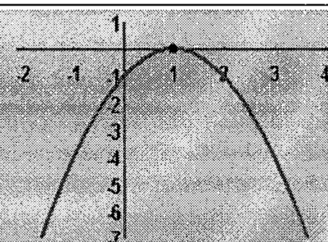
Behavior of a Quadratic Function

Given a quadratic function in the form of **$f(x) = ax^2 + bx + c$**

$a > 0$	$a < 0$
• Opens up • Vertex is a minimum point • ends approach ∞	• Opens down • Vertex is a maximum point • ends approach -∞

x	y
0	4
2	0
4	4



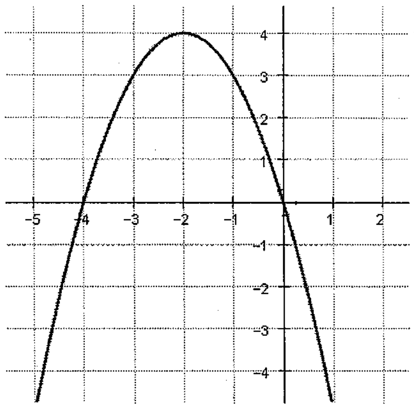


x	y
0	-1
1	0
2	-1

- The function is ***decreasing*** for all values in which **x < 2**
- The function is ***increasing*** for all values in which **x > 2**
- The ends of the graph approach **+ ∞**

- The function is ***increasing*** for all values in which **x < 1**
- The function is ***decreasing*** for all values in which **x > 1**
- The ends of the graph approach **– ∞**

2) Describe the end behavior of the following graphs. Describe the intervals for which the functions are increasing and the intervals for which they are decreasing. State the range of each function.

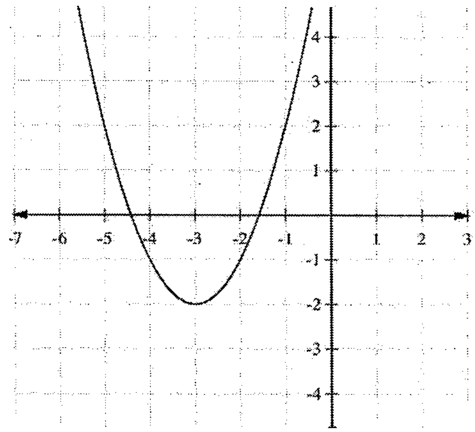


End behavior: ______________

Increasing: ______________

Decreasing: ______________

Range: ______________



End behavior: ______________

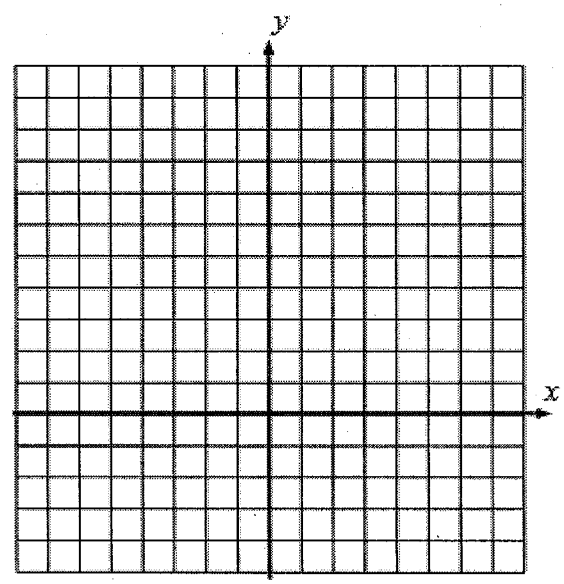
Increasing: ______________

Decreasing: ______________

Range: ______________

3) Consider the quadratic function $f(x) = x^2 + 4x - 1$

(a) Graph the function.



(b) State the range of the function.

(c) State the interval over which $f(x)$ is increasing.

Essential Question: How do we transform a quadratic equation written in standard form to vertex form?

Do Now:

a) Graph $\mathbf{y = -2x^2 + 8x - 6}$ using a table of values.

b) Determine the coordinates of the vertex. ______________

c) State whether the vertex is a *maximum* or a *minimum* point. ______________

d) State and graph the equation of the **axis of symmetry**. ______________

e) State the **roots** of the parabola. ______________

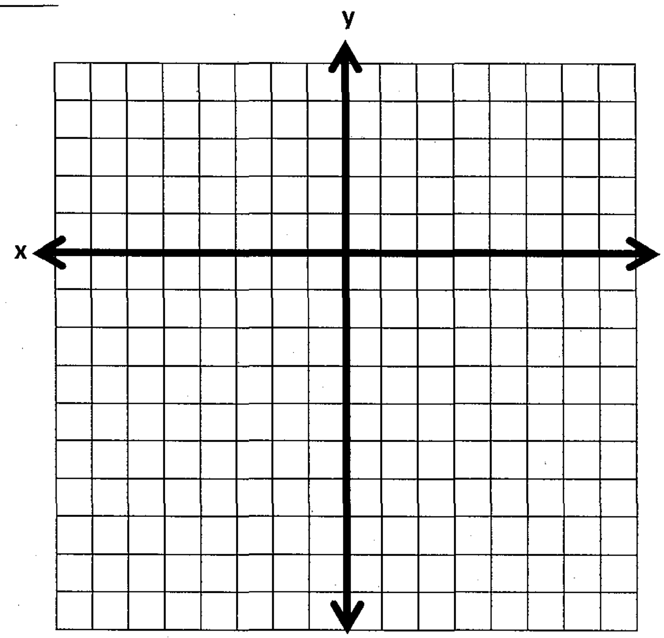
f) State the **y-intercept**. ______________

g) State the **domain** of the function. ______________

h) State the **range** of the function. ______________

i) State the interval for which the function is increasing. __________

j) State the interval for which the function is decreasing. __________



VERTEX FORM OF A QUADRATIC FUNCTION

$$f(x) = a(x - h)^2 + k$$

where ***h*** and ***k*** are real numbers and **(*h*, *k*)** is the vertex

Example: Convert $y = x^2 + 12x + 32$ into vertex form, and state the vertex.

$$y = x^2 + 12x + 32$$
$$y - 32 = x^2 + 12x$$
$$y - 32 + 36 = x^2 + 12x + 36$$
$$y + 4 = x^2 + 12x + 36$$
$$y + 4 = (x + 6)(x + 6)$$
$$y + 4 = (x + 6)^2$$
$$\boxed{y = (x + 6)^2 - 4}$$

Vertex: (−6, −4)

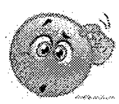
1) Since we will be "completing the square," isolate the x^2 and x terms and move the "c" term to the other side of the equal sign.
2) Find the perfect square trinomial. Take half of the coefficient of the x term, square it, and add it to both sides of the equation.
3) Simplify and factor the perfect square trinomial.
4) Isolate the y term.

Rewrite the following equations in <u>vertex form</u> by completing the square and state the vertex. <u>Check your answer with the table of values on the calculator</u>.

$y = a(x - h)^2 + k$ **Vertex: (h, k)**

1. $y = x^2 + 2x - 4$

2. $y = x^2 - 12x + 4$

Let's try some more complicated examples. 

3. $y = 3x^2 + 18x - 36$

4. $f(x) = -6x^2 - 12x + 48$

A quadratic function written in *standard form* (**$y = ax^2 + bx + c$**) can be rewritten in *vertex form* (**$y = a(x - h)^2 + k$**) by ____. When the function is written in vertex form, the vertex can easily by identified by the ordered pair (______,______). The TAKEAWAY

For each function below written in vertex form, state the vertex of the function.

1) $y = (x + 1)^2 - 7$

2) $y = \frac{1}{2}(x + 4)^2 - 2$

3) $f(x) = 3(x - 1)^2 + 6$

Rewrite each quadratic function in vertex form. State the vertex.

4) $y = x^2 + 10x - 3$

5) $g(x) = -x^2 + 6x - 14$

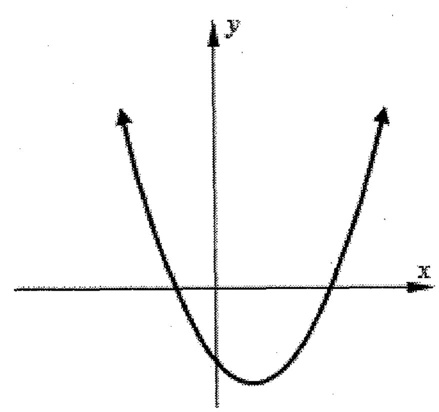
6) **Without using your graphing calculator**, determine which of the following could be the equation of the quadratic shown below. Explain your reasoning.

A. $y = -\frac{1}{2}(x - 2)^2 - 4$

B. $y = -\frac{1}{2}(x + 2)^2 - 4$

C. $y = \frac{1}{2}(x - 2)^2 - 4$

D. $y = \frac{1}{2}(x + 2)^2 - 4$



Essential Questions: In how many ways can we write a quadratic function? What information do the different forms of quadratic functions tell us?

Do Now:

Consider the quadratic equation $y = x^2 + 4x - 12$ *written in standard form.*

a) Rewrite the equation in vertex form.

b) Determine the vertex of the function. __________

Think About This…

Is there another way to write the quadratic function from the Do Now?

Terry says the function $y = x^2 + 4x - 12$ can be written in ***factored form***.

What do you think the function looks like in factored form?

Factored Form ______________________________

What does this equation tell us about the graph of the function?

Standard Form $y = x^2 + 4x - 12$	**Vertex Form** ____________________	**Factored Form** ____________________
• Opens ________ • y-intercept ________	• Opens ________ • Vertex ________	• Opens ________ • Roots ________

Let's Review – **There are three ways we can represent a quadratic function.**

STANDARD FORM

$$f(x) = ax^2 + bx + c$$

where a, b, & c are real numbers

When a quadratic function is written in **standard** form, we find the

- **vertex** by using $x = \frac{-b}{2a}$ to find the x-coordinate. By substituting the x value into the function, we find the y-coordinate of the vertex.
- **roots** by solving the quadratic equation algebraically when f(x) = 0 or by graphing and finding the zeros of the function (locate x-intercepts).
- **y-intercept** by identifying the c value.

VERTEX FORM

$$f(x) = a(x - h)^2 + k$$

where a, h and k are real numbers, (h, k) is the vertex

When a quadratic function is written in **vertex** form, we can determine the

- **vertex** by identifying (h, k) from the equation.

FACTORED FORM

$$f(x) = a(x - r_1)(x - r_2)$$

where a is a real number and r_1 and r_2 are real roots

When a quadratic function is written in **factored** form, we can determine the

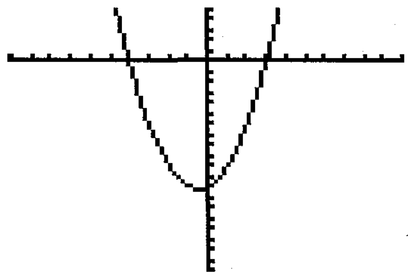
- **roots** by identifying r_1 and r_2 from the equation.

1. The roots for two quadratic functions are given. Write the equation of each function in **factored form** if the a value equals -5.

(a) $r_1 = -2,\ r_2 = 3$

(b) $r_1 = -6,\ r_2 = -1$

2. Write the equation for the function of the graph given below in **factored form** (a = 1).



3. Write the equation for each function in **vertex form** given a and the vertex.

(a) $a = 1$, vertex: (-2, -7)

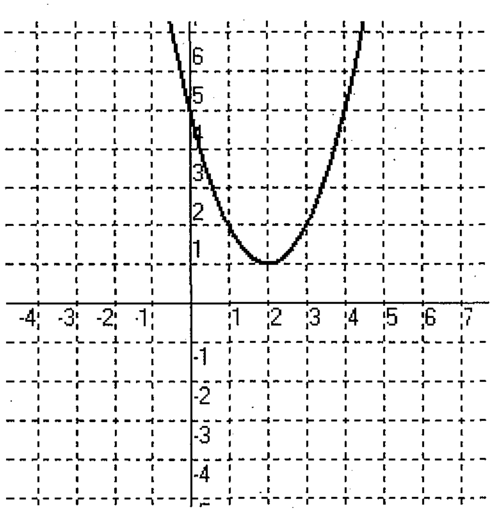
(b) a = -2, vertex: (4, 0)

4. Find the vertex of the following parabolas.

(a) $f(x) = (x - 7)^2 - 4$

(b) $f(x) = 3(x + 4)^2 + 6$

5. Write the equation, in **vertex form**, of the function shown in the graph below if a = 1.



6. Which of the following equations could describe the function seen in the graph at the right? Select all that apply.

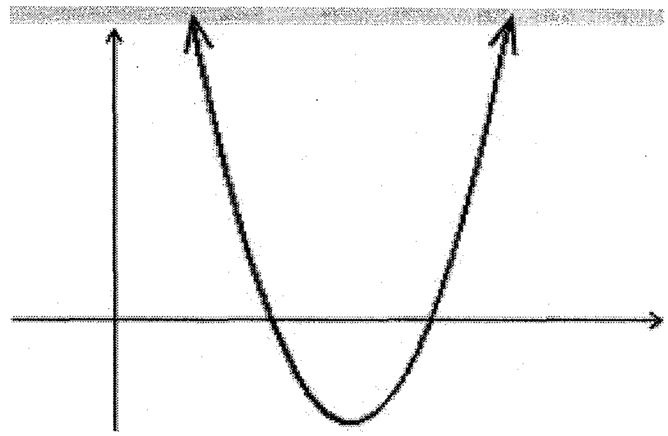
A. $y = (x + 2)(x - 5)$

B. $y = -2x^2 + 4x - 1$

C. $y = (x - 6)(x - 10)$

D. $y = (x + 5)^2 + 4$

E. $y = (x - 8)^2 - 6$



Think about this...

Any equation written in the form $y = a(x^2 + x - 12)$, where a is a constant, has the same solution set as the equation $y = x^2 + x - 12$.

For example, graph the equations $y = x^2 + x - 12$ and $y = 3x^2 + 3x - 36$ on your calculator. What do you notice?

There are three forms in which to write the equation of a quadratic function:

Take Away

- ________________ form: $y =$ ______________________________
- ________________ form: $y =$ ______________________________
- ________________ form: $y =$ ______________________________

8 Algebra CC **HW #** ______________

Fill in the blanks.

1. The ____________________ form of a quadratic function identifies the **turning point**.

2. The ____________________ form of a quadratic function identifies the **roots** (*zeros*).

3. The ____________________ form of a quadratic function identifies the **y-intercept**.

4. For the functions below, complete **a** and **b**.

   a) Is the vertex of the function a ***minimum*** or ***maximum*** value?

   b) State the vertex (*show all necessary work*).

   $y = \frac{1}{5}x^2 - 5x - 1$

   $y = -3(x - 7)^2$

5. Rewrite the function $y = x^2 + 10x - 3$ in ***vertex form*** by completing the square. State the vertex of the function.

6. Rewrite the function $y = 3x^2 - 48$ in ***factored form***. State the zeros of the function.

7. Rewrite the quadratic function $y = -3(x-1)^2 + 5$ in ***standard form***. State the y-intercept of the graph.

8. Write a quadratic function in ***vertex form*** given that $a = 1$ and the vertex is (-3, 4).

9. Write a quadratic function in ***factored form*** given that $a = -10$, $r_1 = -5$ and $r_2 = 9$.

10. For which function below is the zeros of the function -2 and 5?

   A. $f(x) = 4(x-2)(x+5)$

   B. $f(x) = 10x^2 + 30x - 100$

   C. $f(x) = (x-1.5)^2 - 12.25$

   D. $f(x) = (x+2)^2 + 5$

Essential Question: How can we model the path of an object using a quadratic function?

Do Now:

Consider the quadratic function shown in the table below.

x	−1	0	1	2	3	4
y	3	9	11	9	3	-7

Which of the following inequalities represents the range of the function?

(a) $y \geq -7$ (b) $y \leq 4$ (c) $y \geq 3$ (d) $y \leq 11$

Which of the following inequalities represents the interval for which the function is increasing?

(a) $x \geq 1$ (b) $x \leq 1$ (c) $x > 1$ (d) $x < 1$

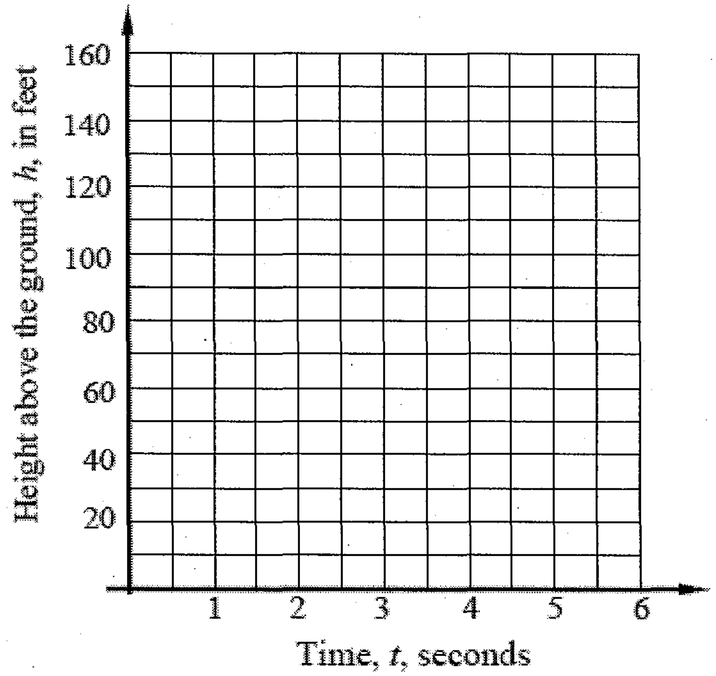
Parabolas in Real Life (https://www.youtube.com/watch?v=He42k1xRpbQ)

If the question asks...	Then calculate the...	How?
When does an object reach its maximum height?		
What is the maximum height of an object?		
How long is an object in the air? When does the object hit the ground?		
What is the initial height of an object?		

1) The height of an object that is traveling through the air can be modeled by a quadratic function that opens downward. The object is fired upward and its height in feet above the ground is modeled by the equation:

$$h(t) = -16t^2 + 64t + 80$$

a) Create a table of values and draw a graph of the object's height for all times where the object is on or above the ground.



b) What is the maximum height the object reaches (*in feet*)?

c) At what time does the object hit the ground?

d) State the domain and range of the function.

e) Over what time interval is the object's height decreasing?

f) Over what time interval is the object's height increasing?

2) A baking soda rocket is fired upwards with an initial speed of 80 feet per second. Its height above the ground, h (*in feet*), can be modeled using the equation, $h(t) = -16t^2 + 80t$, where t is the time since the launch (*in seconds*). At what time does the rocket hit the ground after being launched?

3) A player hits a baseball into the outfield. The equation $h = -0.005x^2 + x + 3$ models the path of the ball, where h is the height and x is the horizontal distance the ball travels.

(a) What is the maximum height reached by the baseball?

(b) An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it?

4) A manufacturer is testing the durability of an object. He decides to throw the object straight up in the air; the height of the object over time can be modeled by the function $f(t) = -16t^2 + 32t + 48$.

(a) State a feasible domain for the above stated function. What does the domain of the function represent in this context?

(b) State a feasible range for the above stated function. What does the range of the function represent in this context?

(c) At what height does the object get thrown from?

(d) After how many seconds does the object hit the ground?

(e) What is the maximum height that the object reaches while in the air? How long does it take for the object to reach this height?

Take Away

The coordinates of the **vertex**, the **y-intercept** and **roots** of a quadratic function help us understand the **parabolic path** of an object.

Turn and Talk

1. A ball is being thrown into the air and follows a path represented by the function $f(t) = -4(t+2)^2 + 36$, where t represents the time since the ball was thrown, in seconds. At what value of t does the ball reach the ground? Justify your response.

2. A flare is launched from the deck of a lifeboat 4 feet above the water surface. The initial velocity is 80 ft/s. The path of the ball can be modeled using the function $h(t) = -16t^2 + 80t + 4$ where t represents the time since the flare was launched, in seconds. *Algebraically*, determine how many seconds it will take for the flare to be 100 feet above the water's surface.

Essential Question: How can we determine if a function is linear, exponential or quadratic?

Do Now:

a) When tables are used to model functions, we typically have just a few sample values of the function and therefore have to do some detective work to figure out what the function might be. What type of function (*linear, exponential or quadratic*) do you think each table models? *Be ready to justify your response.*

x	$f(x)$
0	6
1	12
2	18
3	24
4	30
5	36

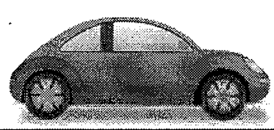
x	$g(x)$
0	0
1	14
2	24
3	30
4	32
5	30

x	$h(x)$
0	1
1	3
2	9
3	27
4	81
5	243

b) Three cars start traveling at the same time. The distance traveled in t minutes is y miles. Graph the distances of each car over the first minute on your calculator. Use the indicated window setting.

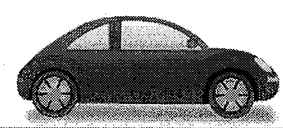
Window Setting
Xmin = 0
Xmax = 1
Xscl = 0.025
Ymin = 0
Ymax = 1
Yscl = 0.025

Car 1



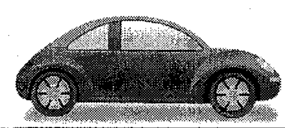
t	$y = t$

Car 2



t	$y = 2^t - 1$

Car 3

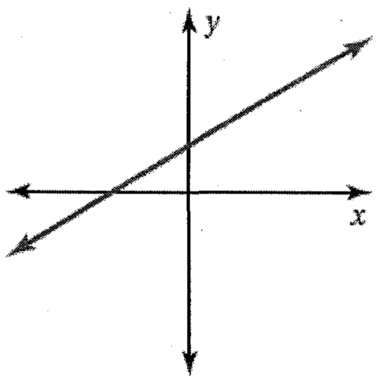


t	$y = t^2$

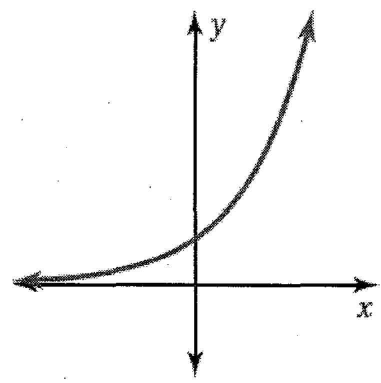
Which car is moving at a constant rate? Explain your reasoning.

Which car accelerated the most during the first minute? Explain your reasoning.

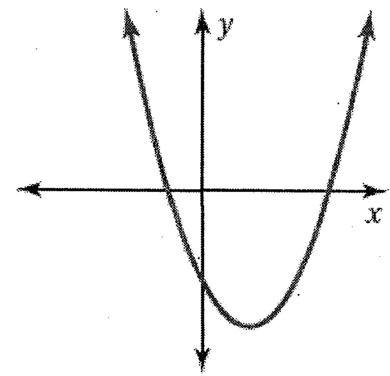
Linear Function | **Exponential Function** | **Quadratic Function**



Line
$y = mx + b$



Curve
$y = ab^x$



Parabola
$y = ax^2 + bx + c$

Identifying Functions Using Differences or Ratios

One method for identifying functions is to look at the ***difference*** or the ***ratio*** of different values of the dependent variable.

If the ***difference*** between values of the dependent variable is the same each time we change the independent variable by the same amount, then the function is **linear**.

+1 +1 +1 +1

x	−2	−1	0	1	2
y	1	3	5	7	9

+2 +2 +2 +2

The y-values have a common *difference* of 2.

If the ***ratio*** between values of the dependent variable is the same each time we change the independent variable by the same amount, then the function is **exponential**.

+1 +1 +1 +1

x	−2	−1	0	1	2
y	1	2	4	8	16

×2 ×2 ×2 ×2

The y-values have a common *ratio* of 2.

Differences can also be used to identify **quadratic** functions. For a quadratic function, when we increase the *x* values by the same amount, the difference between *y* values will *not* be the same. However, the difference of the differences of the *y* values will be the same.

+1 +1 +1 +1

x	−2	−1	0	1	2
y	−1	−2	−1	2	7

−1 +1 +3 +5 ← First differences

+2 +2 +2 ← Second differences

Tell whether the table of values represents a linear, an exponential, or a quadratic function.

1.

x	−2	−1	0	1	2
y	0	0.5	1	1.5	2

2.

x	−1	0	1	2	3
y	0.2	1	5	25	125

3.

x	−2	−1	0	1	2
y	0.75	1.5	3	6	12

4.

x	2	3	4	5	6
y	2	4.5	8	12.5	18

5. Match the function to the situation.

A. $p(x) = -16x^2 + 30x + 160$ B. $f(x) = 10x$ C. $q(x) = 2^x$

_______The population of bacteria doubled every month, and the total population vs. time was recorded.

_______A ball was launched upward from the top of a building, and the vertical distance of the ball from the ground vs. time was recorded.

_______Melvin saves the same amount of money every month. The total amount saved after each month was recorded.

6. Analyze these data sets. Match the function on the right to the table. Use the function to fill in the missing data.

Table A

x	y
0	6
1	10
2	14
3	◯
4	22
5	◯

Table B

x	y
0	6
1	15
2	18
3	15
4	◯
5	◯

Table C

x	y
-1	$\frac{1}{6}$
0	1
1	◯
2	36
3	◯
4	1296

Table D

x	y
-1	◯
0	6
1	8
2	6
3	0
4	◯
5	-24

Equations:

$f(x) = 6^x$

$h(x) = -3(x-2)^2 + 18$

$g(x) = -2(x+1)(x-3)$

$r(x) = 4x + 6$

TAKE AWAY!

How can I tell the difference between linear, exponential and quadratic functions from a table of values?

A **common difference** can be calculated if the function is ________________________________.

A **common ratio** can be calculated if the function is ____________________________________.

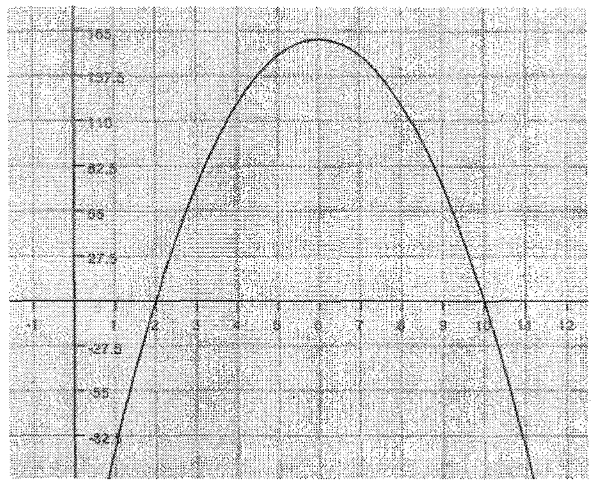
A **common second difference** can be calculated if the function is __________________________.

8 Algebra CC

Unit 15 Extra Practice

For all questions, show all necessary work! ***Justify your responses (provide mathematical evidence)!***

1. A toy company is manufacturing a new toy and trying to decide on a price that will result in a maximum profit. The graph below represents profit (***y***) generated by each price of a toy (***x***). Answer the questions based on the graph of the quadratic function model.



   a) Which domain will result in a profit for the company?

   (1) $2 \leq x \leq 10$ (3) $2 < x < 10$

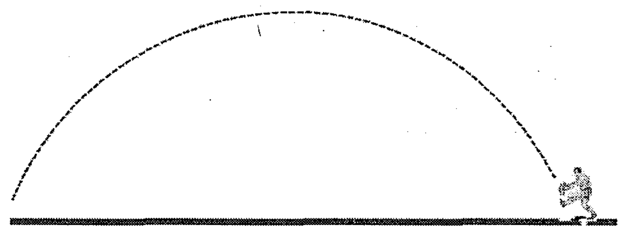
   (2) $0 \leq y \leq 160$ (4) $0 < x < 12$

   b) What part of the graph locates the company's maximum profit?

   (1) The zeroes (3) The y-intercept

   (2) The vertex (4) The domain

2. The height, *in feet*, of a football that is kicked can be modeled by the function $f(x) = -4x^2 + 16x + 2$, where **x** represents the time, *in seconds*, after the ball has been thrown and ***f*(x)** represents the height of the ball.

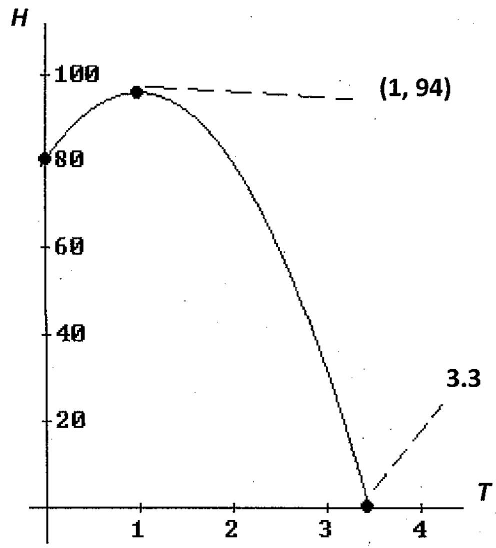


   a) After how many seconds does the ball reach its maximum height?

   b) What is the ball's maximum height?

   c) Rewrite the function in *vertex form*. What information are you given about the football when the equation is rewritten in this form?

3. The height of a launched object is modeled by the graph shown below. ***T*** represents the time, *in seconds*, after the object was launched. ***H*** represents the height of the object, *in feet*, at time ***T***.



a) From what height was the object launched?

b) At what time does the object reach its maximum height?

c) During what time interval is the object's height always *decreasing*?

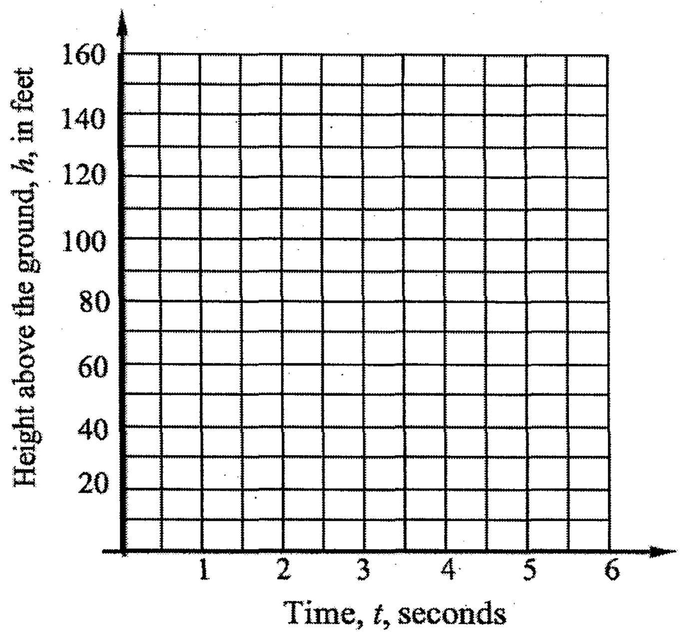
d) State the *domain* of the graph.

e) State the *range* of the graph.

f) What is the *average rate of change* (average speed) of the object during the *first second*?

4. The height of an object above the ground that is fired upward can be modeled by the function $h(t) = -16t^2 + 96t$ where t is the time, in seconds, the object has been in the air.

a) Create a table of values and draw a graph of the object's height for all times where it is at or above the ground.

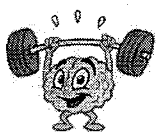


b) What is the maximum height, in feet, of the object?

c) At what time does the object hit the ground?

d) Over what time interval is the height of the object increasing?

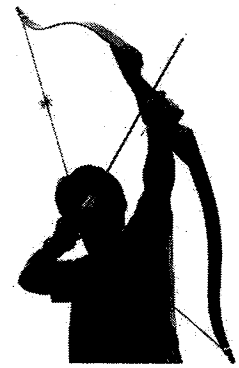
e) At the same time, another object is shot through the air. It follows a linear path. The height of the object above the ground can be modeled by the function: $g(t) = 32t$ where t is the time, in seconds, the object has been in the air. Will the two objects meet? If so, at what time and at what height? *Helpful Hint: Graph the linear function.*



Now that you are warmed up, it's time for a PIP question.

5. An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t, and the height of the arrow in meters, h, is given by

$$h(t) = -4.9t^2 + 29.4t + 2.5.$$



a) What is the maximum height of the arrow?

b) How long does it take the arrow to reach its maximum height?

c) What is the average rate of change for the interval from $t = 1$ to $t = 2$ seconds? Compare your answer to the average rate of change for the interval from $t = 2$ to $t = 3$ seconds, and **explain** the difference in the context of the problem.

d) How long, *to the nearest hundredth of a second*, does it take the arrow to hit the ground? *Helpful Hint: Use the quadratic formula!*

8 Algebra CC
Unit 15 Review (Quadratic Functions)

Important Terminology

quadratic function, parabola, vertex, turning point, axis of symmetry,
maximum, minimum, roots, zeros

What should I be able to do?

1. Graph quadratic functions using a table of values.
2. Determine the coordinates of the turning point of a parabola.
3. Determine whether the y-value of the vertex is a *minimum* or *maximum* value.
4. Identify the roots (zeros) of a quadratic function.
5. State the interval for when a quadratic function is *increasing* or *decreasing*.
6. State the domain and range of a quadratic function.
7. Rewrite quadratic functions in *standard form*, *vertex form* and *factored form*.
8. Answer questions about the parabolic path of an object.

Practice Problem Set

1. a) Graph $y = -x^2 - 5x + 3$ using a table of values.

   b) State and graph the equation of the **axis of symmetry.**_______________

   c) Determine the coordinates of the **turning point.**_______________

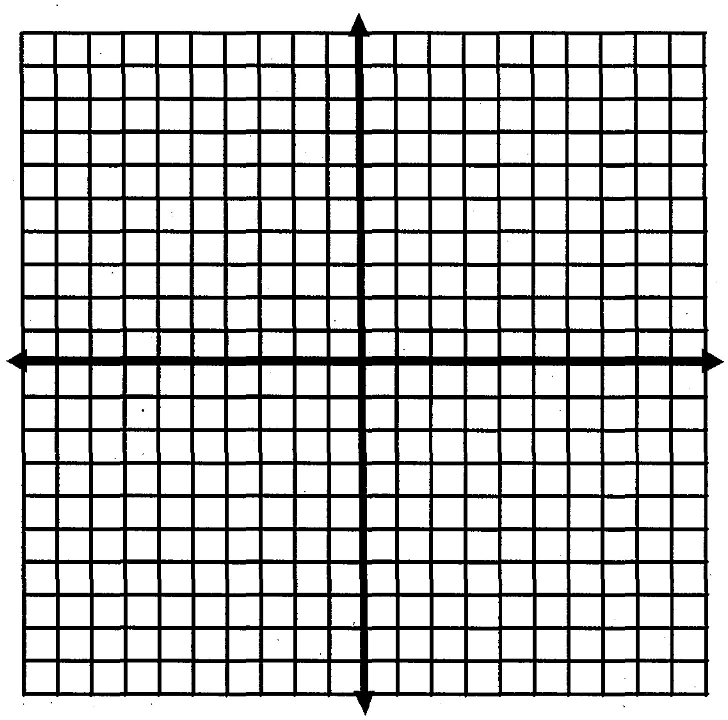
   d) State whether the vertex is a *maximum* or a *minimum* point._______________

   e) State the **roots** of the parabola (round to the nearest hundredth)._______________

   f) State the **y-intercept** of the graph._______________

   g) State the **range** of the function. _______________

x	y



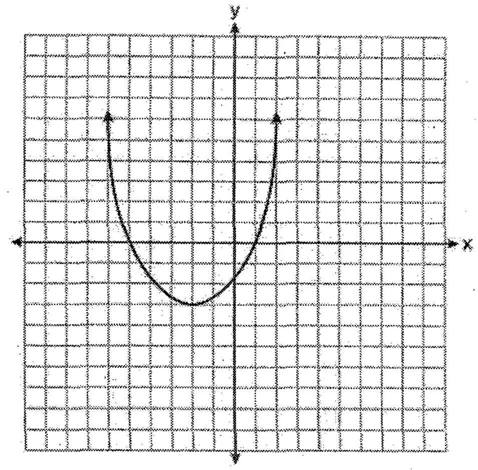
2) Let f be the function represented by the graph.

a) State the **roots** of the function.__________

b) State the **vertex**. __________

c) Let g be a function such that $\mathbf{g(x) = \frac{1}{2}x^2 + 4x + 3}$.

Determine which function has the smaller *minimum* value. Justify your response.

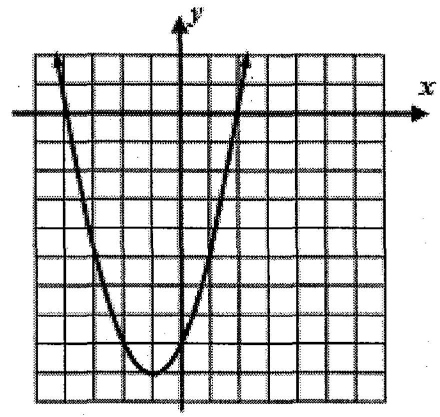


3) Each quadratic function below has a *domain* of all real numbers. State the ***range*** of each function.

$h(x) = -(x + 2)^2 + 5$

x	y
-3	1
-2	-5
-1	-7
0	-5
1	1

$f(x) = 4x^2 + 8x - 6$

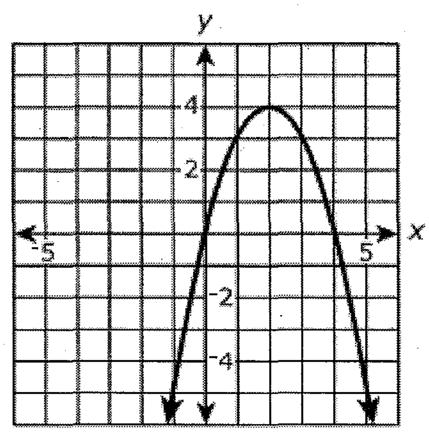


4) The graph of the function $\mathbf{f(x) = 4x - x^2}$ is shown here.

a) State the ***range*** of the function.

b) State the interval on which $f(x)$ is ***increasing***.

c) State the interval on which $f(x)$ is ***decreasing***.



5) State the **zeros** of the function $\mathbf{f(x) = -10(x + 3)(x - 7)}$.

6) In the xy-coordinate plane, the graph of the equation $\mathbf{y = 3x^2 - 12x - c}$ has zeros at x = a and x = b, where a < b. The graph has a minimum at (2, -48). What are the values of a, b and c?

7) Given the function ***f*(x) = $x^2 + 6x + 8$**

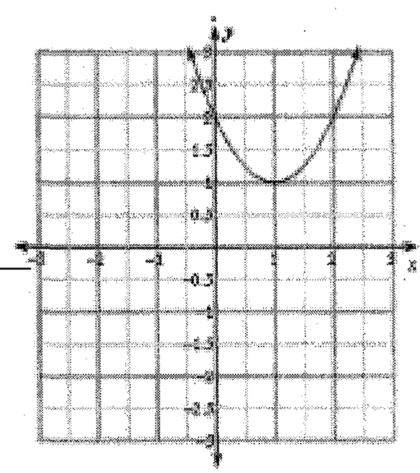
a) Rewrite the function in **factored form**. State the **zeros** of the function.

b) Rewrite the function in **vertex form** by completing the square. State the **vertex** of the function.

c) Rewrite the function **y = $2x^2 - 8x + 6$** in **vertex form** by completing the square. State the vertex of the function.

d) Which function has the smaller *minimum* value?

8) Answer **a** and **b** based on the graph shown here.

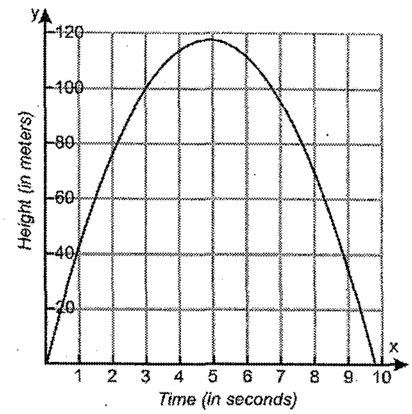


a) Are the roots *real* or *non-real* numbers? ________________

b) Is the discriminant ($b^2 - 4ac$) *positive* or *negative*? ________________

9) The height of an object after it has been launched is modeled by the graph of the quadratic function shown here where **y** represents the height of the object from the ground after **x** seconds.

Calculate the *average rate of change* of the height of the object for the first 3 seconds after being launched.



10) A student throws a bag of chips to her friend. Unfortunately, her friend does not catch the chips, and the bag hits the ground. The distance from the ground (height) for the bag of chips is modeled by the function $\boldsymbol{h(t) = -16t^2 + 32t + 4}$, where $\boldsymbol{h}$ is the height (distance from the ground in feet) of the chips, and $\boldsymbol{t}$ is the number of seconds the chips are in the air.

a) From what height are the chips being thrown?

b) What is the maximum height the bag of chips reaches while airborne?

c) How long does it take the bag of chips to reach its maximum height?

11) A rocket is launched from a cliff. The relationship between the height of the rocket, *in feet*, and the time since its launch, ***t***, *in seconds* can be represented by the function $\boldsymbol{h(t) = -16t^2 + 80t + 384}$. How long did it take for the rocket to hit the ground?

8 Algebra CC HW #

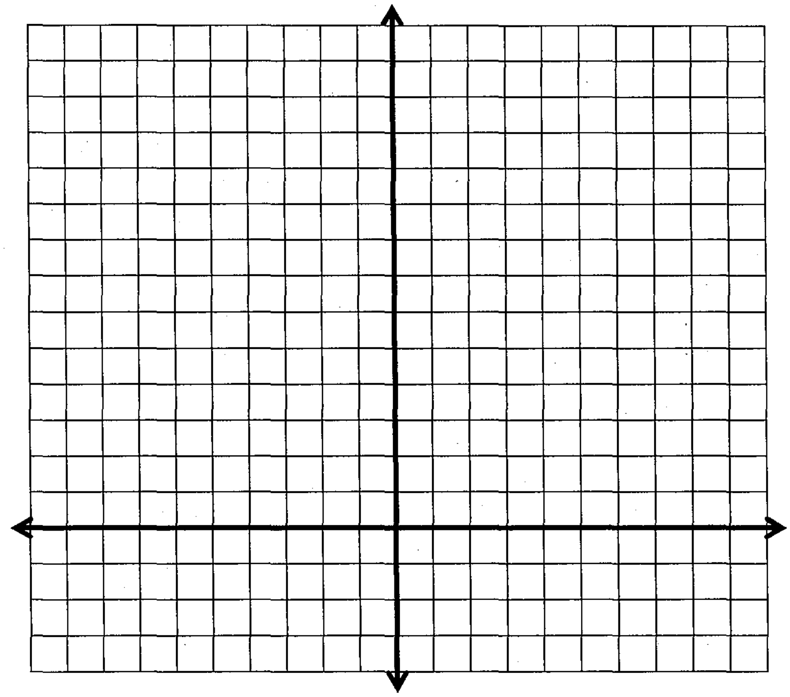
FLIP VIDEO LESSON (rmsalgebra.weebly.com)

Solving Linear-Quadratic Systems Graphically

A linear quadratic system contains a linear equation and a quadratic equation: $\begin{Bmatrix} y = mx + b \\ y = ax^2 + bx + c \end{Bmatrix}$

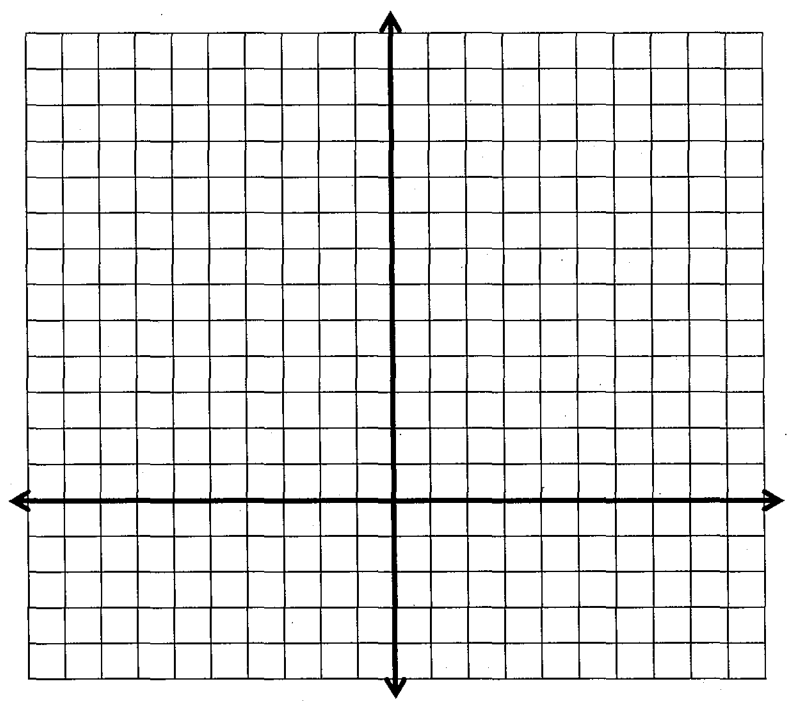
1. $y = x^2$ $\quad$ $2y - 2x = 12$

x	y



2. $y = -x^2 + 2x + 8$ $\quad$ $y = 9$

x	y



3. Solve the system below using your graphing calculator.

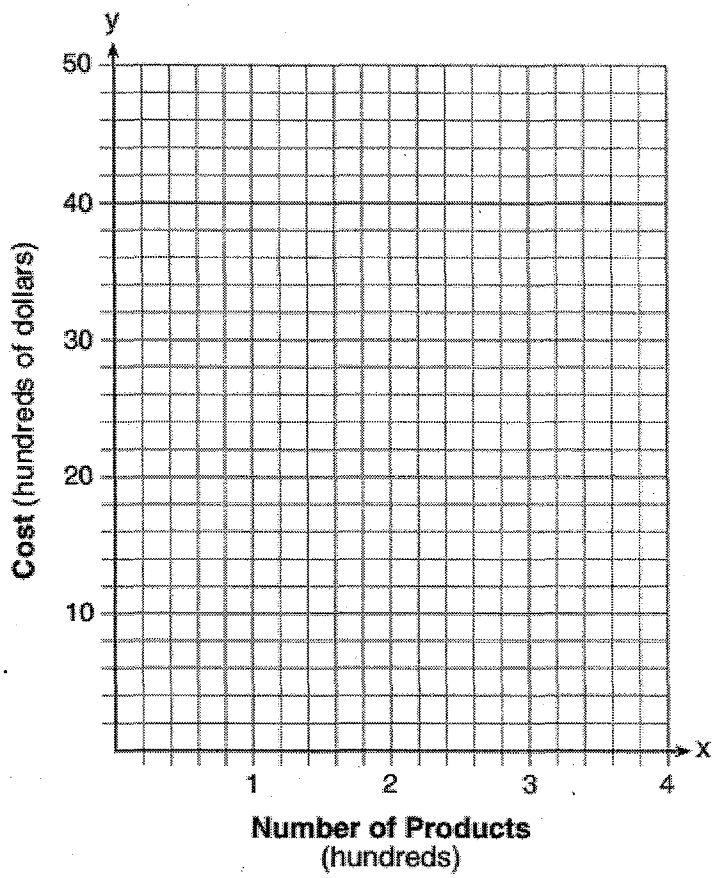
$$y = -\frac{1}{2}x^2 - 3x + 1 \qquad y = -\frac{1}{3}x + 6$$

Let's try a system in the context of a situation.

4. A company is considering building a manufacturing plant. They determine the weekly production cost at site A to be **A(x) = 3x²** while the production cost at site B is **B(x) = 8x + 3**, where **x** represents the number of products, in hundreds, and **A(x)** and **B(x)** are production costs, in hundreds of dollars.

   a) Graph the production cost functions on the set of axes below and *label them site A and site B*.

x	A(x)	B(x)



   b) State the positive value(s) of x for which the production costs at the two sites are equal.

THE TAKE AWAY...

A system consisting of a quadratic equation and a linear equation can have _______,

_______ or _______ solutions.