NAME _____ DATE _____

Unit 14 Review Algebra RH

Solve each quadratic equation using the *square root method.*

1. $4x^2 = 64$

$x^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

2. $25x^2 = 324$

$x^2 = \frac{324}{25}$

$\sqrt{x^2} = \pm\sqrt{\frac{324}{25}}$

$x = \pm \frac{18}{5}$

3. $x^2 - 12 = 0$

$x^2 = 12$

$\sqrt{x^2} = \pm\sqrt{12}$

$x = \pm\sqrt{4}\sqrt{3}$

$x = \pm 2\sqrt{3}$

4. $x^2 + 9 = 0$

$x^2 = -9$

$\sqrt{x^2} = \pm\sqrt{-9}$

$x = \pm 3i$

Solve each quadratic equation by *factoring.*

5. $x(x-1) - 6 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x-3=0 \mid x+2=0$

$x = 3 \mid x = -2$

$\{-2, 3\}$

6. $5x^2 + 15x = 0$

$5x(x+3) = 0$

$5x=0 \mid x+3=0$

$x=0 \mid x=-3$

$\{-3, 0\}$

7. $2x^2 = 18x - 28$

$2x^2 - 18x + 28 = 0$

$x^2 - 9x + 14 = 0$

$(x-7)(x-2) = 0$

$x-7=0 \mid x-2=0$

$x = 7 \mid x = 2$

8. $2x^2 + x - 15 = 0$ $AC = -30$ $B = 1$

$2x^2 - 6x + 5x - 15 = 0$

$2x(x-3) + 5(x-3) = 0$

$(2x+5)(x-3) = 0$

$2x+5=0 \mid x-3=0$

$2x = -5 \mid x = 3$

$x = -\frac{5}{2}$

not necessary to write in set notation

Solve each quadratic equation using the *quadratic formula.* If necessary, express answers in simplest radical form.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ set = to zero first!

9. $2x^2 + 5x + 3 = 0$

$a=2$ $b=5$ $c=3$

$x = \frac{-5 \pm \sqrt{(5)^2-4(2)(3)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{25-24}}{4}$

$x = \frac{-5 \pm \sqrt{1}}{4}$

$\frac{-5+1}{4}$ $\frac{-5-1}{4}$

$x = -1, -\frac{3}{2}$

10. $-3x^2 - 10x = 5$

$-3x^2 - 10x - 5 = 0$

$a=-3$ $b=-10$ $c=-5$

$x = \frac{10 \pm \sqrt{(-10)^2-4(-3)(-5)}}{2(-3)}$

$x = \frac{10 \pm \sqrt{100-60}}{-6}$

$x = \frac{10 \pm \sqrt{40}}{-6} = \frac{10 \pm 2\sqrt{10}}{-6}$

$x = \frac{5+\sqrt{10}}{-3}, \frac{5-\sqrt{10}}{-3}$

11. $x^2 = 6x - 25$

$x^2 - 6x + 25 = 0$

$a=1$ $b=-6$ $c=25$

$x = \frac{6 \pm \sqrt{(-6)^2-4(1)(25)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36-100}}{2}$

$x = \frac{6 \pm \sqrt{-64}}{2}$

$x = \frac{6 \pm 8i}{2} = 3 \pm 4i$

$x = 3+4i, 3-4i$

Solve the quadratic equation by *completing the square.* If necessary, express answers in simplest radical form.

12. $x^2 + 10x - 23 = 0$

$x^2 + 10x = 23$

$\left(\frac{10}{2}\right)^2$

$x^2 + 10x + 25 = 23 + 25$

$(x+5)^2 = 48$

$\sqrt{(x+5)^2} = \pm\sqrt{48}$

$x + 5 = \pm 4\sqrt{3}$

$x = -5 + 4\sqrt{3}, -5 - 4\sqrt{3}$

13. $2x^2 - 20x + 32 = 0$

$x^2 - 10x + 16 = 0$

$\left(\frac{-10}{2}\right)^2$ $x^2 - 10x = -16$

$x^2 - 10x + 25 = -16 + 25$

$\sqrt{(x-5)^2} = \pm\sqrt{9}$

$x - 5 = \pm 3$

$x = 5 + 3, 5 - 3$

$x = 8, 2$

14. $x^2 - 9x = -8$

$\left(\frac{-9}{2}\right)^2$

$x^2 - 9x + 20.25 = -8 + 20.25$

$(x - 4.5)^2 = 12.25$

$\sqrt{(x-4.5)^2} = \pm\sqrt{12.25}$

$x - 4.5 = \pm 3.5$

$x = 4.5 + 3.5, 4.5 - 3.5$

$x = 8, 1$

Solve each quadratic using *any method.* If necessary, express answers in simplest radical form.

→ your work may look different, depending on the method. The solutions should be the same.

15. $\frac{x-4}{x-5} = \frac{x}{3}$

$x(x-5) = 3(x-4)$

$x^2 - 5x = 3x - 12$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x - 6 = 0$ | $x - 2 = 0$

$x = 6$ | $x = 2$

16. $(x-2)(x-4) = 6$

$x^2 - 6x + 8 = 6$

$\left(\frac{-6}{2}\right)^2$ $x^2 - 6x = -2$

$x^2 - 6x + 9 = -2 + 9$

$\sqrt{(x-3)^2} = \pm\sqrt{7}$

$x - 3 = \pm\sqrt{7}$

$x = 3 + \sqrt{7}, 3 - \sqrt{7}$

For each quadratic, find the discriminant and describe the nature of the roots.

→ $b^2 - 4ac$

17. $x^2 - 4x + 5 = 0$

$a = 1 \quad b = -4 \quad c = 5$

$b^2 - 4ac$

$(-4)^2 - 4(1)(5)$

$16 - 20$

-4

imaginary (two non-real solutions)

18. $4x^2 = -4x - 1$

$4x^2 + 4x + 1 = 0$

$a = 4 \quad b = 4 \quad c = 1$

$b^2 - 4ac$

$(4)^2 - 4(4)(1)$

$16 - 16$

0

real, rational, equal (one distinct solution)

19. $3x^2 + 5x = -1$

$3x^2 + 5x + 1 = 0$

$a = 3 \quad b = 5 \quad c = 1$

$b^2 - 4ac$

$(5)^2 - 4(3)(1)$

$25 - 12$

13

real, irrational, unequal (2 distinct solutions)

20. $x^2 = 8x$

$x^2 - 8x = 0$

$a = 1 \quad b = -8 \quad c = 0$

$b^2 - 4ac$

$(-8)^2 - 4(1)(0)$

64

real, rational, unequal (two distinct solutions)

21. If the quadratic $3x^2 - 2x + k = 0$ has *one real root*, find the value of k.

discriminant = 0

$a = 3$, $b = -2$, $c = k$

$b^2 - 4ac = 0$

$(-2)^2 - 4(3)(k) = 0$

$4 - 12k = 0$

$-12k = -4$

$k = 1/3$

22. If the quadratic $kx^2 + 4x + 1 = 0$ has *imaginary roots*, find all possible values of k.

discriminant < 0

$a = k$, $b = 4$, $c = 1$

$b^2 - 4ac < 0$

$(4)^2 - 4(k)(1) < 0$

$16 - 4k < 0$

$-4k < -16$

$k > 4$

23. Given the quadratics below, determine the roots (zeros).

when the graph of the quadratic function crosses the x-axis (the y coordinate will always be 0)

a. $f(x) = (x - 5)(x + 2)$

$0 = (x-5)(x+2)$

$x - 5 = 0$ | $x + 2 = 0$

$x = 5$ | $x = -2$

b. $y = x^2 - 9x - 36$

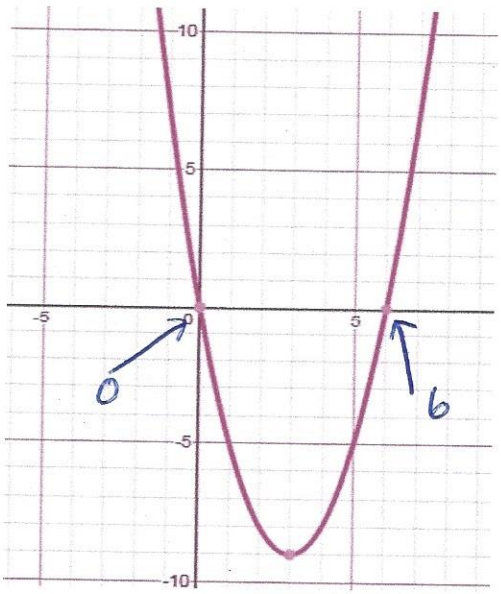
$0 = x^2 - 9x - 36$

$0 = (x-12)(x+3)$

$x - 12 = 0$ | $x + 3 = 0$

$x = 12$ | $x = -3$

c.



$x = 0, 6$

24. Write the quadratic in *factored form* and then state the roots.

$y = x^2 + 2x - 15$

$y = (x+5)(x-3)$

roots: $-5, +3$

25. Given the quadratics below, determine the vertex (turning point).

a. $y = 2(x-6)^2 - 5$

$(6, -5)$

b. $f(x) = x^2 + 8x + 17$

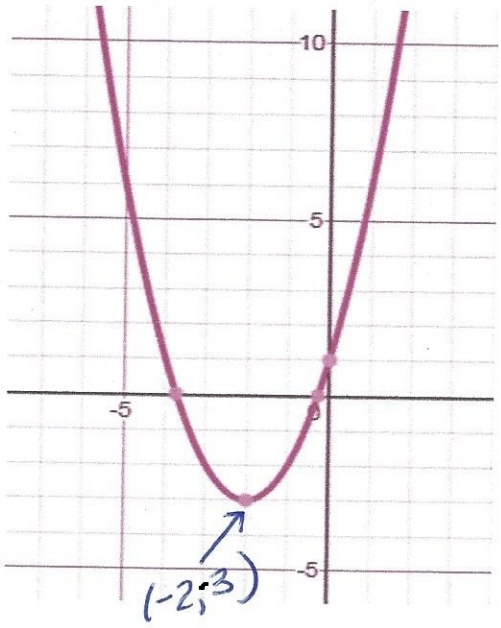
$x = \frac{-b}{2a}$ $\quad y = (-4)^2 + 8(-4) + 17$

$x = \frac{-8}{2(1)}$ $\quad y = 16 - 32 + 17$

$x = -4$ $\quad y = 1$

$(-4, 1)$

c.



26. Write the following quadratic in *vertex form* (using complete the square method) and then state the vertex.

$y = x^2 + 2x - 4$

$\left(\frac{2}{2}\right)^2$

$y + 4 = x^2 + 2x$

$y + 4 + 1 = x^2 + 2x + 1$

$y + 5 = (x+1)^2$

$y = (x+1)^2 - 5$

or

$y = x^2 + 2x \quad -4$

$y = x^2 + 2x + 1 \quad -4 - 1$

$y = (x+1)^2 - 5$

vertex: $(-1, -5)$

27. Determine the equation of the *axis of symmetry* of the quadratic below:

$y = -4x^2 - 2x + 9$

$x = \frac{-b}{2a}$

$x = \frac{-(-2)}{2(-4)}$

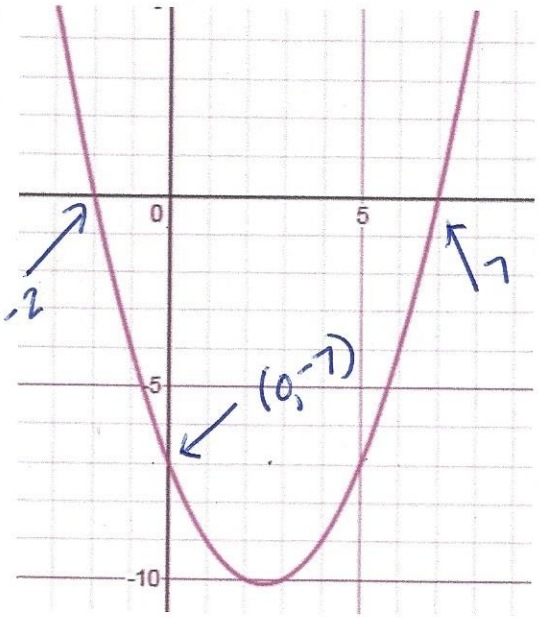
$x = -\frac{1}{4}$

28. Given the following information, write the equation of the quadratic in *factored form.*

$r_1 = 5, r_2 = -8, a = -4$

$y = -4(x-5)(x+8)$

29. Write the equation of the quadratic below in *factored form*.



$y = a(x+2)(x-7)$

$-7 = a(0+2)(0-7)$

$-7 = a(2)(-7)$

$-7 = -14a$

$\frac{-7}{-14} = a$

$a = \frac{1}{2}$

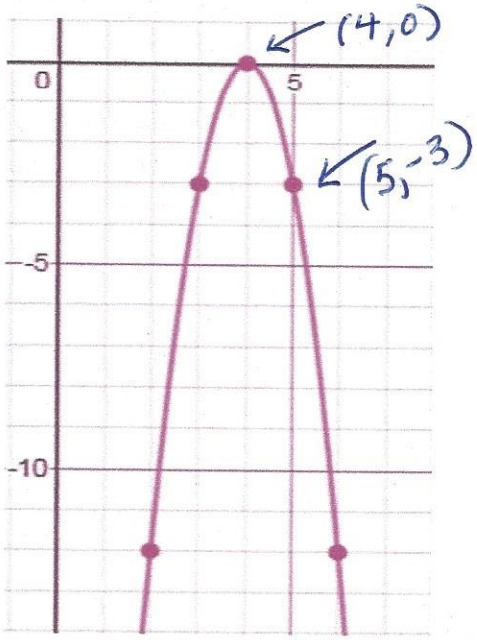
$y = \frac{1}{2}(x+2)(x-7)$

30. Given the following information, write the equation of the quadratic in *vertex form.*

Vertex: $(17, 5)$, $a = \frac{1}{4}$

$y = \frac{1}{4}(x-17)^2 + 5$

31. Write the equation of the quadratic below in *vertex form*.



$y = a(x-4)^2 + 0$

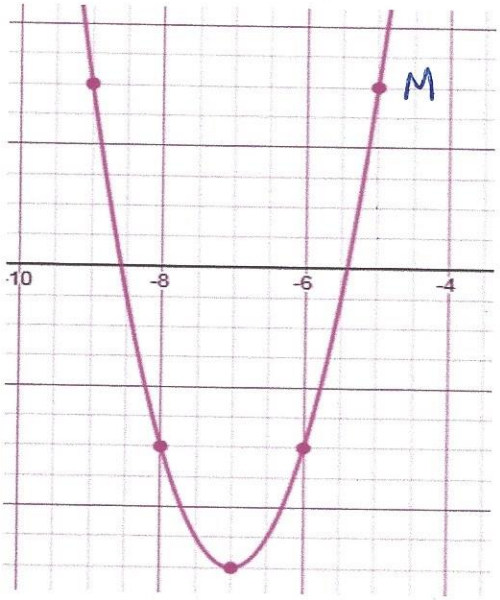
$-3 = a(5-4)^2$

$-3 = a(1)^2$

$-3 = a$

$y = -3(x-4)^2$

32. Write the equation of the parabola below in ***standard form.***



$y = 4x^2 + 56x + 186$

if you counted by 1 on the y-axis

vertex $(-7,-10)$

$y = a(x+7)^2 - 10$

$M(-5,6)$

$6 = a(-5+7)^2 - 10$

$6 = a(+2)^2 - 10$

$6 = 4a - 10$

$16 = 4a$

$a = 4$

$y = 4(x+7)^2 - 10$

$y = 4(x+7)(x+7) - 10$

if you counted by 2 on the y-axis

vertex $(-7,-5)$

$y = a(x+7)^2 - 5$

$M(-5,3)$

$3 = a(-5+7)^2 - 5$

$3 = a(2)^2 - 5$

$3 = 4a - 5$

$8 = 4a$

$a = 2$

$y = 2(x+7)^2 - 5$

$y = 2(x+7)(x+7) - 5$

$y = 2(x^2+14x+49) - 5$

$y = 2x^2 + 28x + 98 - 5$

$y = 2x^2 + 28x + 93$

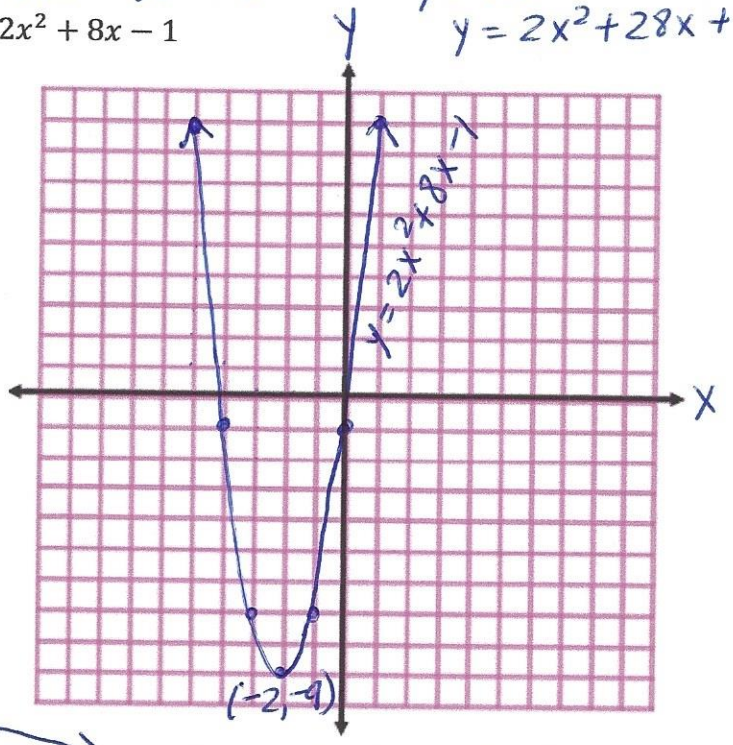
33. Graph and label the quadratic function $y = 2x^2 + 8x - 1$

x	y
-5	9
-4	-1
-3	-7
-2	-9
-1	-7
0	-1
1	9

$x = \frac{-b}{2a}$

$x = \frac{-8}{2(2)}$

$x = -2$



a. Vertex: $(-2,-9)$

b. Maximum or minimum? minimum

c. Roots (Zeros): $\frac{-4 \pm 3\sqrt{2}}{2}$

d. Domain: $(-\infty, \infty)$

e. Range: $[-9, \infty)$

x values

f. Interval where increasing: $(-2, \infty)$

g. Interval where decreasing: $(-\infty, 2)$

Show work for the roots here:

$y = 2x^2 + 8x - 1$

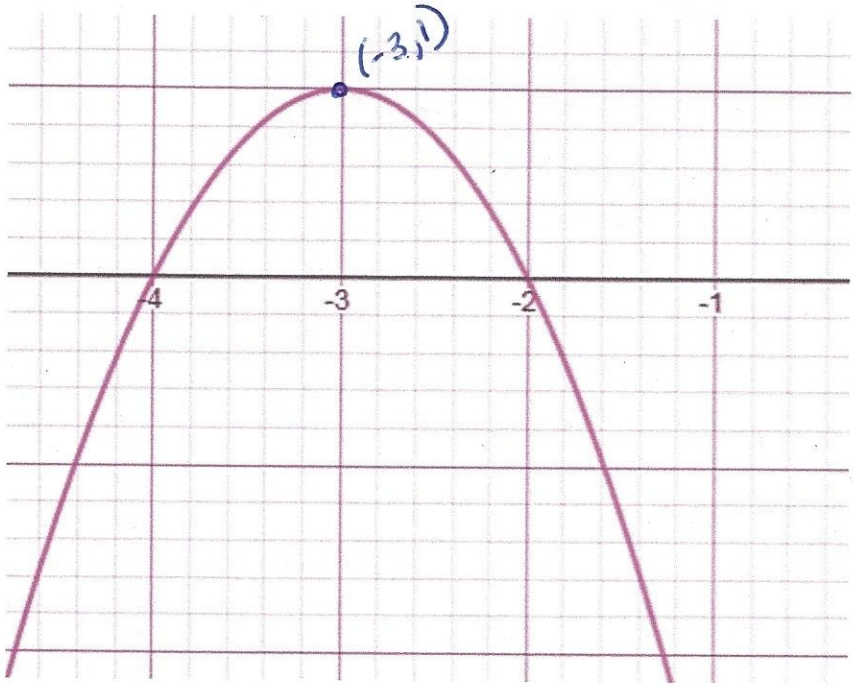
$a = 2 \quad b = 8 \quad c = -1$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(-1)}}{2(2)}$

$x = \frac{-8 \pm \sqrt{72}}{4} = \frac{-8 \pm \sqrt{36}\sqrt{2}}{4}$

$= \frac{-8 \pm 6\sqrt{2}}{4} = \frac{-4 \pm 3\sqrt{2}}{2}$

34. Given the following graph below, state the:



a. End Behavior: approach $-\infty$

b. Increasing: $(-\infty, -3)$ $x < -3$

c. Decreasing: $(-3, \infty)$ $x > -3$

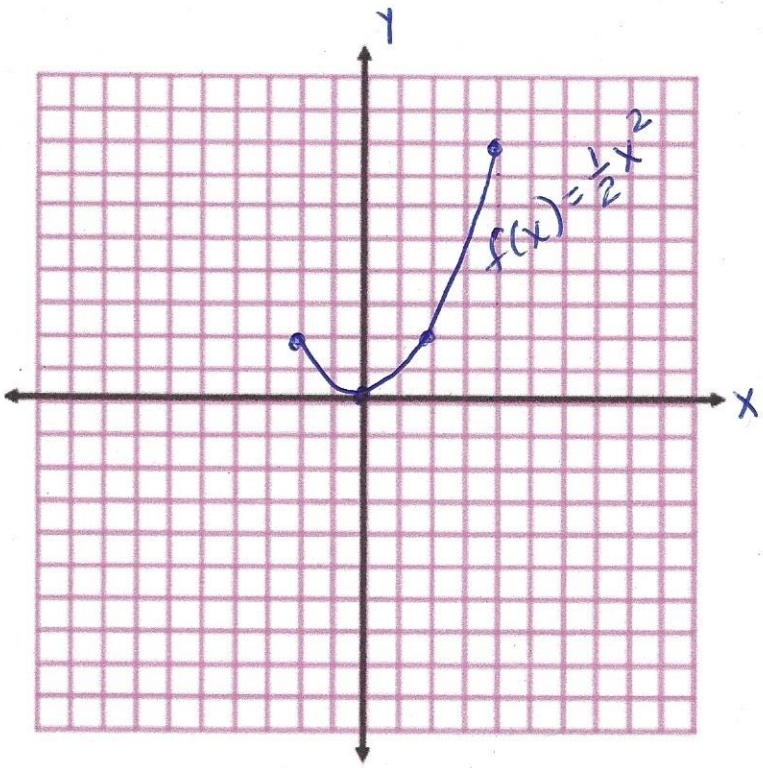
d. Domain: $(-\infty, \infty)$ $x \in \mathbb{R}$

e. Range: $(-\infty, 1]$ $y \leq 1$

35. For the quadratic function $f(x) = \frac{1}{2}x^2$ defined on the interval $-2 \leq x \leq 4$

no arrows

Graph the function for the stated interval



x	y
-2	2
0	0
2	2
4	8

a. State the range of the function

$0 \leq y \leq 8$

$[0, 8]$

b. State the interval on which f(x) is increasing

$4 > x > 0$ or $0 < x < 4$

$(0, 4)$

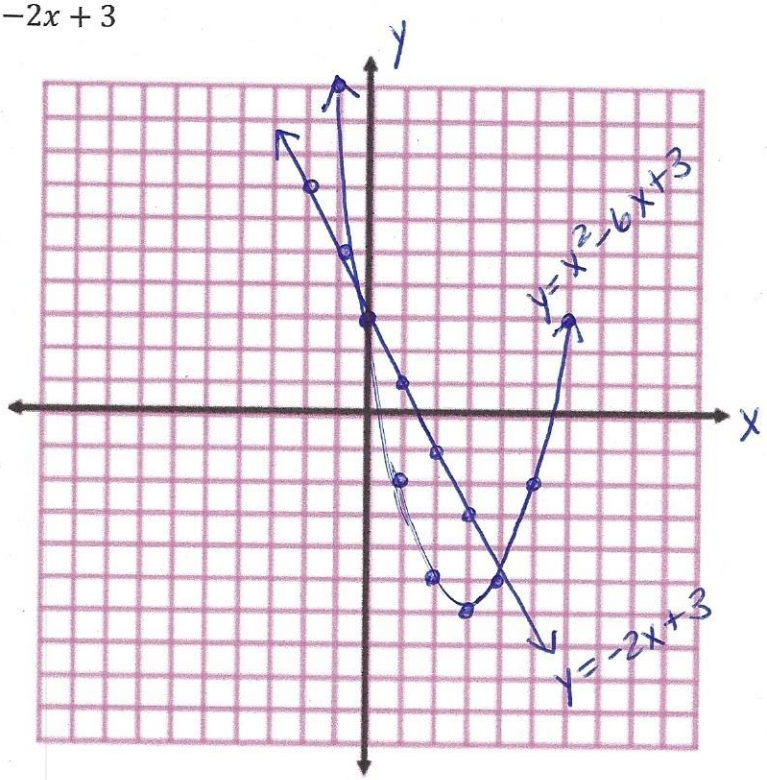
c. State the interval on which f(x) is decreasing

$-2 < x < 0$

$(-2, 0)$

36. Solve the following quadratic linear systems *graphically*

a. $y = x^2 - 6x + 3$
$y = -2x + 3$



$y = x^2 - 6x + 3$

$x = \frac{-b}{2a} = \frac{6}{2(1)} = 3$

x	y	
-1	10	
0	3	
1	-2	
2	-5	
3	-6	← vertex
4	-5	
5	-2	
6	3	

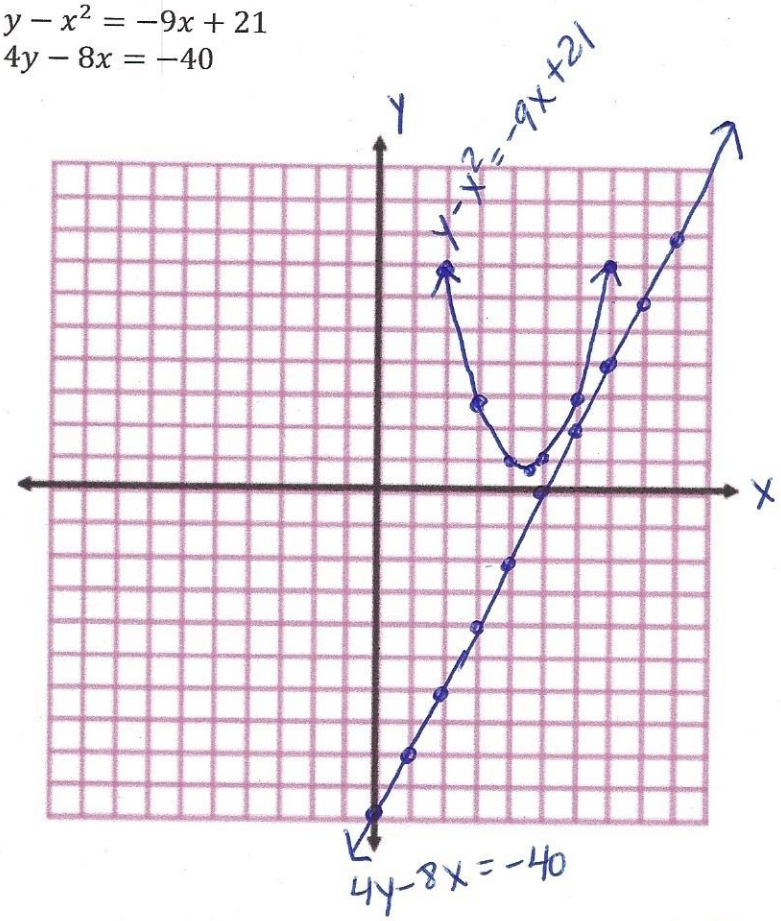
$y = -2x + 3$

$m = -\frac{2}{1}$ $b = 3$

common solutions

(0, 3) (4, -5)

b. $y - x^2 = -9x + 21$
$4y - 8x = -40$



$y = x^2 - 9x + 21$

$x = \frac{-b}{2a} = \frac{9}{2(1)} = 4.5$

x	y	
2	7	
3	3	
4	1	
4.5	.75	← vertex
5	1	
6	3	
7	7	

$y = (4.5)^2 - 9(4.5) + 21$

$y = .75$

$4y - 8x = -40$

$4y = 8x - 40$

$y = 2x - 10$

$m = \frac{2}{1}$ $b = -10$

no solutions

37. Solve the following linear quadratic systems *algebraically.*

substitution method

remember to get values of both ordered pairs

a. $y = x^2 - x - 6$
$y = 2x - 2$

$2x - 2 = x^2 - x - 6$

$x^2 - 3x - 6 = -2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x - 4 = 0$	$x + 1 = 0$
$x = 4$	$x = -1$

$y = 2x - 2$

$y = 2(4) - 2$
$y = 8 - 2$
$y = 6$

$y = 2(-1) - 2$
$y = -2 - 2$
$y = -4$

common solutions
(4,6) (-1,-4)

check:

$y = x^2 - x - 6$
$6 = 4^2 - 4 - 6$
$6 = 6$ ✓
$y = 2x - 2$
$6 = 2(4) - 2$
$6 = 6$ ✓

$y = x^2 - x - 6$
$-4 = (-1)^2 - 1(-1) - 6$
$-4 = 1 + 1 - 6$
$-4 = -4$ ✓
$y = 2x - 2$
$-4 = 2(-1) - 2$
$-4 = -4$ ✓

b. $y = x^2 - 3x - 4$
$y = x - 8$

$x - 8 = x^2 - 3x - 4$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x - 2 = 0$	$x - 2 = 0$
$x = 2$	$x = 2$

$y = x - 8$
$y = 2 - 8$
$y = -6$

$(2, -6)$

check (2,-6)

$y = x^2 - 3x - 4$
$-6 = (2)^2 - 3(2) - 4$
$-6 = 4 - 6 - 4$
$-6 = -6$ ✓

$y = x - 8$
$-6 = 2 - 8$
$-6 = -6$ ✓

tangent to a curve
only intersects in one place