UNIT 14

Solving

Quadratic

Equations

Essential Question: How do we solve quadratic equations?

Do Now: Compare and contrast the equations below.

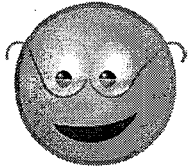a) $x^2 + 1 = 10$ b) $x + 1 = 9$

<table>
<tr><td>Think about this to help you…
•   Are the equations equivalent?
•   Would you solve the equations in the same way?
•   Do the equations have the same number of solutions?</td></tr>
</table>

Quadratic Equation: _____

Let's look at another quadratic equation. How would you solve $x^2 - 6x + 8 = 0$?

Examples:

1) $x^2 - 8x = -16$

2) $x^2 + 5x = 36$

3) $x^2 - 16 = 0$

4) $4x^2 - x = 0$

5) $3x^2 - 6x - 45 = 0$

6) $5x^2 - 125 = 0$

Solving Quadratic Equations by Factoring

1) Rewrite the equation in the form of $ax^2 + bx + c = 0$

2) Factor

3) Set each factor equal to zero and solve (*zero product property*)

4) Check solution set with the original equation

7) $x(x-2) = 35$

8) $x^2 + 5x - 12 = 8x - 2$

TAKE AWAY

Quadratic Equations can be solved by _____ and using the _____ _____ property. If the product of two quantities equals zero, at least one of the quantities must equal zero.

One more question...

The solution set of the equation $x^2 - 4x - 12 = 0$ is

(1) {-6,2}   (3) {-2,6}

(2) {-4,3}   (4) {-3,4}

Solve the following quadratic equations.

1. $x^2 - 3x + 2 = 0$

2. $z^2 - 5z + 4 = 0$

3. $x^2 - 8x + 16 = 0$

4. $c^2 + 6c = -5$

5. $10m^2 + 10m = 0$

6. $m^2 - 64 = 0$

7. $3x^2 - 12 = 0$

8. $2x^2 + 20x = -18$

9. $5x^2 - 60x = 140$

Essential Question: How do we solve quadratic equations?

Do Now: Solve the quadratic equation below.

$$3x^2 - 48 = 0$$

Think about this...

Is there another way to solve the quadratic equation from the Do Now?

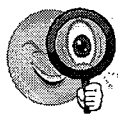| Solving Quadratic Equations by Taking Square Roots ($ax^2 + c = 0$) |
| --- |
| 1) Isolate x^2 |
| 2) Take the square root of both sides of the equation |
| 3) Check solution set with the original equation |

$$3x^2 - 48 = 0$$

Examples:

1) $x^2 = 64$

2) $x^2 - 36 = 0$

3) $3x^2 = 300$

4) $2x^2 - 30 = 68$

5) $x^2 + 5 = 8$

6) $10x^2 - 20 = 0$

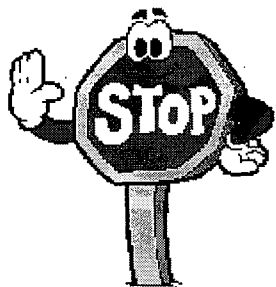Let's take a look at some more complex quadratic equations.

7) $(d+4)^2 = 16$

8) $(y-5)^2 = 49$

9) $3(x+1)^2 + 2 = 29$

10) $4(m-9)^2 = 52$

Quadratic Equations in the form of $ax^2 + c = 0$ can be solved by taking the _____ _____. Quadratic equations in the form of $ax^2 + bx + c = 0$ are solved by _____ and using the zero product property.

Reminder:
The **square root method** only works for quadratic equations in the form of **ax² + c = 0**.
Quadratics in the form of **ax² + bx + c = 0** can only be solved by **factoring**.

Solve each quadratic below.

1. $x^2 + 2x - 8 = 0$

2. $6x^2 - 24x = 72$

3. $4(x - 3)^2 = 20$

4. $5x^2 + 20x = 0$

5. $9x^2 = 81$

6. $\dfrac{x}{9} = \dfrac{2}{x - 3}$

Essential Question: How can quadratic equations help us solve problems?

Do Now: A landscaper is creating a rectangular flower bed such that the width is half the length. The area of the flower bed is 32 square feet. Using **x** to represent the width, write an equation that can be used to find the width of the flower bed.

Let's take a closer look at the equation from the Do Now and solve. What is the width of the flower bed?

Let's try some more examples.

1) Mary is six years older than her cousin Joan. The product of their ages is 135. Find their ages.

2) Find two consecutive *negative* integers such that the square of the first decreased by 17 equals 4 times the second.

3) An elementary school is designing a set of square garden plots so that each grade can grow its own vegetables. The minimum size for a plot recommended for vegetable gardening is at least 2 meters on each side. The school principal has decided to make the vegetable gardens bigger by adding an additional **x** meters to each side.

a. Write an expression to represent the area of one garden.

b. There are 6 grades in the school including pre-kindergarten and kindergarten. Write an expression to represent the total area of all 6 gardens.

c. The total area available for the gardens is 150 square meters. Write and solve an equation to calculate the dimensions of each square garden.

TAKE AWAY! | Quadratic Equations can help us solve problems. It's important to analyze the solution set to the equation and determine which solution(s) make sense in the context of the situation.

For each problem below, solve by setting up an equation. Make sure to define all unknowns with variable expressions.

1) The length of a rectangle is 4 times its width. If the area of the rectangle is 256 in^2, find the length and width of the rectangle.

2) One number is 10 less than another number. The product of the two numbers is -25. Find both numbers.

3) When the first of three *positive* consecutive integers is multiplied by the third, the result is one less than six times the second. Find the integers.

Essential Question: How can quadratic equations help us solve more complicated problems?

1. The Smiths have decided to put a paved border of uniform width around their swimming pool. The pool is a rectangular shape that measures 12 feet by 20 feet. The area of the border is 68 ft^2 and the width of the border is **x** feet.
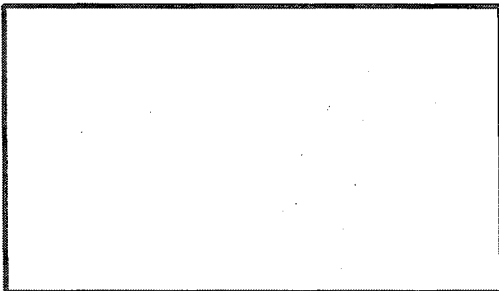
a. Label the diagram to represent the scenario presented above.
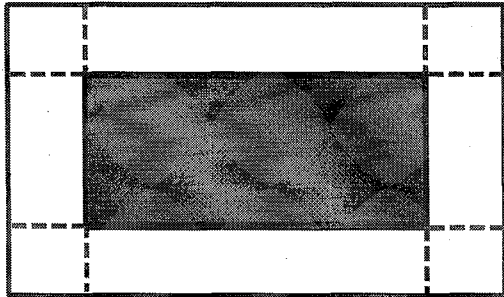
b. What are the dimensions of the small rectangle? What is the area of the small rectangle?
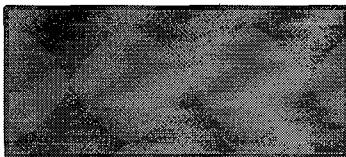
c. What are the dimensions of the large rectangle? What is the area of the large rectangle?

d. Write an equation that represents the area of the large rectangle. Solve the equation. What does the value of the variable represent?

2. A museum is displaying Egyptian artifacts in a 34 by 10 foot rectangular area. To protect the artifacts, a roped-off border has been created around the display. The combined area of the display and the border totals 640 square feet. Find the width of the border.

a. Draw a diagram to represent the scenario. Let **x** represent the width of the border.

b. Represent the dimensions of the large rectangle algebraically.

length:

width:

c. Write an equation that represents the combined area of the display and the surrounding border. Solve the equation to find the value of x.

Essential Question: What is the quadratic formula and how can it help us solve quadratic equations?

Do Now: Solve the quadratic equation: $x^2 + 7x + 6 = 0$

Up until this point, you have solved quadratic equations in <u>two ways</u>. You have solved quadratic equations in the form of $ax^2 + c = 0$ by finding the **square root** and you have solved quadratic equations in the form of $ax^2 + bx + c = 0$ by **factoring**.

Think about this...
Is there another way to solve a quadratic equation?

The **quadratic formula**, derived from $ax^2 + bx + c = 0$, is a formula that can be used to solve **any** quadratic equation.

**Quadratic Formula:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when $a \neq 0$

Let's use the formula to solve the quadratic equation from the Do Now. $x^2 + 7x + 6 = 0$

1) Rewrite as $ax^2 + bx + c = 0$ and identify a, b and c.

2) Write the quadratic formula.

3) Substitute the identified values for a, b and c into the formula.

4) Simplify the expression under the square root symbol ($\sqrt{}$). Simplify the denominator.

5) Evaluate the square root (if possible).

6) Write solutions as two equations.

7) Simplify both solutions.

Think about this....

Can you solve the quadratic equation $x^2 + 2x - 1 = 0$ by factoring?
If you can't factor, what method can you use to solve the equation?
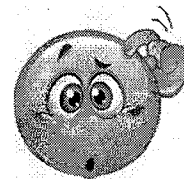
What do you notice about the solutions to the quadratic equation? Is there a way to simplify the solutions?

Quick Review: Let's simplify the following radical expressions (*irrational numbers*).

1. $\pm\sqrt{8}$ 　　　　　2. $\pm\sqrt{27}$ 　　　　　3. $\pm\sqrt{32}$ 　　　　　4. $\pm\sqrt{10}$

Let's simplify more complicated radical expressions (*irrational numbers*).

5. $\dfrac{4 \pm 6\sqrt{3}}{2}$ 　　　　　6. $\dfrac{10 \pm 8\sqrt{7}}{4}$ 　　　　　7. $\dfrac{12 \pm \sqrt{24}}{2}$ 　　　　　8. $\dfrac{8 \pm \sqrt{18}}{2}$

Now let's take a closer look at the irrational solutions from $x^2 + 2x - 1 = 0$.

Solve the quadratic equation below and express your answer in simplest radical form.

$x^2 - 4 = 2x$

The quadratic formula can be used to solve **any** quadratic equation. However, it is most useful when solving quadratic equations that cannot be _____.

In how many ways can $x^2 - 4 = 0$ be solved?
Name every method that can be used.
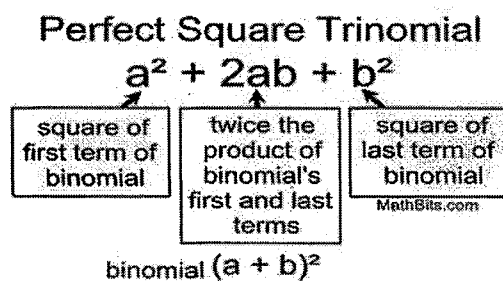
Essential Question: How do we solve quadratic equations by completing the square?

Do Now:

a) Simplify $(x - 5)(x - 5)$ b) Factor $x^2 + 8x + 16$

The simplified polynomial expressions in the Do Now are <u>Perfect Square Trinomials</u>:

Perfect Square Trinomial
$$a^2 + 2ab + b^2$$

| square of first term of binomial | twice the product of binomial's first and last terms | square of last term of binomial |

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binomial $(a + b)^2$

Find the value of c that makes each quadratic trinomial a perfect square.

a) $x^2 + 6x$ _____   b) $x^2 - 2x$ _____   c) $x^2 - 14x$ _____

We can solve Quadratic Equations using a method called <u>Completing The Square</u>. It requires creating perfect square trinomials.

1. Be sure that the coefficient of the highest power is one. If it is not, divide each term by that value to create a leading coefficient of one.	$x^2 + 8x - 4 = 0$
2. Move the constant term to the right hand side.	$x^2 + 8x = 4$
3. Prepare to add the needed value to create the perfect square trinomial. Be sure to balance the equation. The boxes may help you remember to balance.	$x^2 + 8x + \Box = 4 + \Box$
4. To find the needed value for the perfect square trinomial, take half of the coefficient of the *middle term* (x-term), square it, and add that value to both sides of the equation. Take half and square $x^2 + 8x + \Box = 4 + \Box$	$x^2 + 8x + \boxed{16} = 4 + \boxed{16}$
5. Factor the perfect square trinomial.	$(x + 4)^2 = 20$
6. Take the square root of each side and solve. Remember to consider both plus and minus results.	$x + 4 = \pm\sqrt{20}$ $x = -4 \pm \sqrt{20} = -4 \pm 2\sqrt{5}$ $x = -4 + 2\sqrt{5}$ $x = -4 - 2\sqrt{5}$

Solve each quadratic equation below by completing the square.

(1) $x^2 - 6x - 14 = 0$

(2) $x^2 + 2x - 7 = 0$

The "a" value must always be 1! Divide all terms by the "a" value before completing the square.

(3) $2x^2 + 16x = 4$

(4) $-x^2 + 8x = -9$

TAKE AWAY

There is another method we can use to solve a quadratic equation. It involves

dividing _____ by two, squaring the result and _____ that value to

_____ sides of the equation. This is known as _____

_____.

Unit 14 Review – Quadratic Equations

Terminology and Formulas

Quadratic Equation	Complete The Square	Radical Expression	Solution Set

Quadratic Formula $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

What should I be able to do?
- Solve quadratic equations by:
  - ➢ finding the square root
  - ➢ factoring
  - ➢ using the quadratic formula
  - ➢ completing the square
- Problem solve using quadratic equations
- Simplify radical expressions (*irrational solutions*) that result from solving quadratic equations

Practice Problem Set

1) Solve each equation by either taking the **square root** or by **factoring**.

   a) $3x^2 = 27$

   b) $2x^2 - 12x = -16$

   c) $x^2 - 5x = 0$

   d) $8(x - 4)^2 = 200$

   e) $x(x + 3) = 40$

   f) $\dfrac{x}{5} = \dfrac{3}{x + 2}$

2) Find two consecutive negative integers such that the square of the smaller increased by twice the larger is 37.

3) The length of a rectangle is 4 inches more than twice the width. If the area of the rectangle is 48 in^2, find the dimensions of the rectangle.

4) An 8 inch by 10 inch photo is surrounded by a frame of uniform width (x inches). If the area of the photo and the frame is 120 square inches, find the width of the frame.
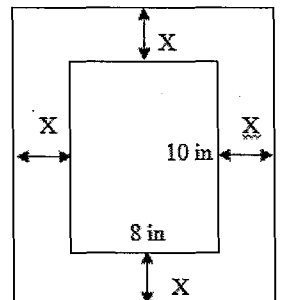
5) Use the quadratic formula to solve $2x^2 - 6x + 3 = 0$. Express your answer in simplest radical form.

6) Complete the square in order to solve $x^2 + 6x - 6 = 0$. Express your answer in simplest radical form.

Unit 14 Practice

Let's Review – Use your notes to help you solve the equations below.

| Solve $-2x^2 + 24x - 46 = 0$ by **completing the square**.	Solve $-x^2 + 5 = 2x$ using the **quadratic formula**.
$$-2x^2 + 24x - 46 = 0$$	$$-x^2 + 5 = 2x$$

Now that you know all the different ways to solve a quadratic equation, decide as a group which method to use to solve each equation on the following page. *Pick the most efficient method for each problem.*

Methods for Solving Quadratic Equations:

1) Square Root $(ax^2 + c = 0)$

2) Factor $(ax^2 + bx + c = 0)$

3) Quadratic Formula

4) Complete the Square

Don't FORGET!
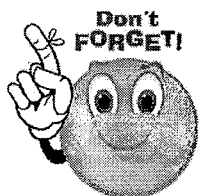
*When quadratic equations cannot be solved by factoring, use the **quadratic formula** or **complete the square** to solve.*

All irrational solutions must be expressed in <u>simplest radical form</u>.

1. $2x^2 + 18x = -16$
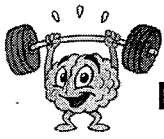
2. $3x^2 + 5 = 152$

3. $4x^2 + 4x - 9 = 0$

4. $10x^2 + 2x = 0$

5. $x^2 - 10x + 15 = 8$

6. $6x^2 = 300$

7. Meredith is deciding which method to use to solve $5x^2 - 7x - 6 = 0$. Jeremy says that she should complete the square and Greg says she should use the quadratic formula. Who do you agree with? <u>Explain your reasoning and justify your response</u>.

Now that you are warmed up, it's time for some PIPS questions!

8. A student was given the equation $x^2 + 6x - 13 = 0$ to solve by completing the square. The first step that was written is shown below.

   $x^2 + 6x = 13$

   The next step in the student's process was $x^2 + 6x + c = 13 + c$.

   State the value of **c** that creates a perfect square trinomial. **Explain** how the value of **c** is determined.

9. Sally and Jane have ages that are consecutive even integers. The product of their ages is 168. Write an equation that can be used to find Sally's age, **s**, if she is the oldest. Solve your equation and state the age of Sally.

10. The dimensions of a rectangular garden are 5 meters by 12 meters. In order to create a bigger rectangular garden, each dimension was increased by the same amount. The new garden's area is double that of the original garden. Find the dimensions of the new garden. *Draw a diagram to help you.*

11. A decorator places a rug in a 9 meter by 12 meter room. A uniform strip of flooring around the rug remains uncovered. Write and solve an equation to determine the width, *x meters*, of the strip of flooring if the area of the rug is half the area of the room. Find the length and width of the rug.
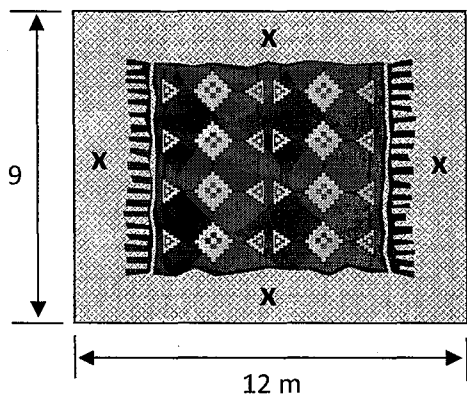
Helpful Hint: *The key to solving this problem is setting up an equation that represents the <u>area of the rug</u>. First, represent the length and width of the rug algebraically.*

9

12 m

12. New Clarendon Park is undergoing renovations to its gardens. One garden that was originally a square is being adjusted so that one side is doubled in length, while the other side is decreased by three meters.

Part A: Represent the dimensions (*length and width*) of the rectangular garden algebraically. Use the diagrams below to help you.

Let **x** = the side length of the original square garden.
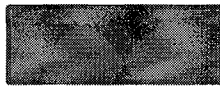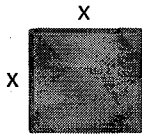
Part B: Represent the **area** of the original *square* garden as a simplified polynomial expression in terms of **x**.

Part C: Represent the **area** of the *rectangular* garden as a simplified polynomial expression in terms of **x**.

Part D: The new rectangular garden will have an area that is 25% more than the original square garden. Write an equation that can be used to determine the length of a side, **x**, of the original square garden. Solve your equation and state the side length of the original square garden.
See the helpful hint box below to help you make sense of a 25% increase.

Helpful Hint: Think about what percent a new amount represents after it has been increased by 25%.

Example:
Original Amount: **10**
25% Increase: .25(10) = **2.5**
New Amount: 10 + 2.5 = **12.5**

New Amount = 125% of the Original Amount
New Amount = 1.25(10)
New Amount = **12.5**

$$\begin{array}{r} 10.0 \leftarrow 100\% \\ + \ 2.5 \leftarrow 25\% \\ \hline 12.5 \leftarrow 125\% \end{array}$$

If **p** is increased by 25% then **1.25p** represents the new amount after a 25% increase.