# **ARITHMETIC AND GEOMETRIC SEQUENCES**

#### **ARITHMETIC SEQUENCES**

- 1) What is the common difference for the following sequence? 14, 12.25, 10.5, 8.75, ...
- 2) Each term in a sequence of numbers is 7 less than the previous term. If the second term of the sequence is 28, write the equation that can be used to find the *n*th term of the sequence.

#### **GEOMETRIC SEQUENCES**

- 3) What is the *common ratio* for the following sequence?  $-\frac{2}{5}$ ,  $\frac{1}{10}$ ,  $-\frac{1}{40}$ , ...
- 4) Which sequence of numbers listed below displays a geometric sequence?

5) Write the first four terms of the geometric sequence, given  $a_1 = 18$  and  $r = -\frac{1}{2}$ 

# **ARITHMETIC AND GEOMETRIC SEQUENCES**

1) To find the common difference, take one term and subtract the previous term

$$12.25 - 14 = -1.75$$
  $10.5 - 12.25 = -1.75$ 

$$d = -1.75$$

2) Second term is 28 so the first term is 7 more than that, 35 35, 28, 21, 14 arithmetic sequence  $a_1 = 35 d = -7$ 

$$a_n = 35 - 7(n-1)$$

### **GEOMETRIC SEQUENCES**

3) To find the common ratio, take one term and divide it by the previous term

$$\frac{1}{10} \div \frac{2}{5} = \frac{1}{4} \qquad \frac{1}{40} \div \frac{1}{10} = \frac{1}{4}$$

$$-\frac{1}{40} \div \frac{1}{10} = -\frac{1}{4}$$

$$\mathbf{r} = -\frac{1}{4}$$

4) Which sequence of numbers listed below displays a geometric sequence?

(arithmetic d = 1.25)

(geometric 
$$r = \frac{1}{2}$$
)

5) Write the first four terms of the geometric sequence, given  $a_1 = 18$  and  $r = -\frac{1}{2}$ 

Keep multiplying by 
$$-\frac{1}{2}$$
  $a_2 = 18 \cdot -\frac{1}{2} = -9$ 

$$a_2 = 18 \cdot - \frac{1}{2} = -9$$

$$a_3 = -9 \cdot -\frac{1}{2} = 4.5$$

$$a_4 = 4.5 \bullet - \frac{1}{2} = -2.25$$

first 4 terms are: 18, -9, 4.5, -2.25

#### IS IT ARITHMETIC OR GEOMETRIC?

1) Determine if the sequence is arithmetic or geometric and identify the next term in the sequence.

- 2) Find the 17th term of the following sequence: 13, 17, 21, 25, ...
- 3) A sequence has the following terms:  $a_1 = 6$ ,  $a_2 = 9$ ,  $a_3 = 13.5$ ,  $a_4 = 20.25$ . Which formula represents the  $n^{th}$  term in the sequence?

A) 
$$a_n = 6 + 1.5n$$
 C)  $a_n = 6(1.5)^n$ 

C) 
$$a_n = 6(1.5)^n$$

B) 
$$a_n = 6 + 1.5(n-1)$$
 D)  $a_n = 6(1.5)^{n-1}$ 

D) 
$$a_n = 6(1.5)^{n-1}$$

4) Write an *explicit* formula for the *n*th term of the sequence shown below

$$a_{\mathsf{n}} = -11, \, -2, \, 7, \, 16, \, \dots$$

#### IS IT ARITHMETIC OR GEOMETRIC?

1) 
$$.3 \div .75 = .4$$

$$.12 \div .3 = -4$$

Geometric sequence r = .4

Next term is .048 • .4 = .0192

2) Find the 17th term of the following sequence: 13, 17, 21, 25, ... This is an arithmetic sequence. d = 4  $a_1 = 13$ 

> Explicit rule:  $a_n = 13 + 4(n-1)$ To find  $17^{th}$  term, n = 17 $a_{17} = 13 + 4(17 - 1)$  $a_{17} = 77$

3) A sequence has the following terms:  $a_1 = 6$ ,  $a_2 = 9$ ,  $a_3 = 13.5$ ,  $a_4 = 20.25$ . Which formula represents the  $n^{th}$  term in the sequence? This is a geometric sequence with r = 1.5

A) 
$$a_n = 6 + 1.5n$$

C) 
$$a_n = 6(1.5)^n$$

B) 
$$a_n = 6 + 1.5(n - 1)$$

B) 
$$a_n = 6 + 1.5(n-1)$$
 D)  $a_n = 6(1.5)^{n-1}$ 

4) Write an explicit formula for the nth term of the sequence shown below

$$a_n = -11, -2, 7, 16, ...$$

arithmetic sequence

$$a_1 = -11$$
  $d = 9$ 

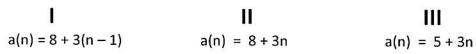
$$a_n = -11 + 9(n - 1)$$

#### IS IT ARITHMETIC OR GEOMETRIC?

1) A pattern of dots is shown below.

First Group Second Group Third Group

If the pattern of dots continues, which formula(s) can be used to determine the number of dots in the nth group?



Number of Years	i	2	2	1
n	1	2	3	4
Money in Account $a(n)$	\$550	\$605	\$665.50	\$732.05

- a) Write an explicit rule that defines the sequence displayed by the table.
- b) If the pattern continues, how much money will be in the account after 5 years?
- c) The person who is saving money can go on a world trip when the balance in the account reaches at least \$1000. In how many years can the person go on the trip?
- 3) After a rock concert ends, the number of people that still remain in the stadium after *n* minutes is displayed by the table below.

Minutes n	1	2	3	4
People a(n)	10,456	9,954	9,452	8,950

- a) Write an explicit rule that models the table.
- b) If the pattern continues, how many people will be in the stadium 14 minutes after the concert ends?
- c) How many minutes have gone by if there are 6,942 people in the stadium?

#### IS IT ARITHMETIC OR GEOMETRIC?

1) First Group

8

Second Group

Third Group

11

14

Arithmetic sequence  $a_1 = 8$  d = 3

$$a_1 = 8 d =$$

explicit rule:  $a_n = 8 + 3(n - 1)$ 

$$8 + 3n - 3$$

Simplified:  $a_n = 5 + 3n$ 

$$a_n = 5 + 3n$$

I and III

)	Number of Years	1	2	2	4
	n	1	2	3	4
	Money in Account	\$550	\$605	\$665.50	\$732.05
	a(n)	Φ230	\$005	\$005.50	\$132.03

a) Write an *explicit* rule that defines the sequence displayed by the table.

Geometric r = 1.1

$$a_n = 550(1.1)^{n-1}$$

b) If the pattern continues, how much money will be in the account after 5 years?

$$a_5 = 550(1.1)^{5-1} = $805.26$$

c) The person who is saving money can go on a world trip when the balance in the account reaches at least \$1000. In how many years can the person go on the trip?  $1000 = 550(1.1)^x$  put the equation  $y = 550(1.1)^x$  into the calculator

Find the table of values where the y-value is closest to \$1000

a(14) = 3930

In 8 years

3)

Minutes n	1	2	3	4
People $a(n)$	10,456	9,954	9,452	8,950

a) Write an explicit rule that models the table. Arithmetic a(1) = 10,456 d = -505

$$a(n) = 10,456 - 502(n - 1)$$

b) If the pattern continues, how many people will be in the stadium 14 minutes after the concert

$$a(14) = 10456 - 502(14 - 1)$$

ends?

c) How many minutes have gone by if there are 6,942 people in the stadium?

$$6942 = 10456 - 502(n - 1)$$

$$6942 = 10456 - 502n + 502$$

$$6942 = 10958 - 502n$$

8 minutes

# **RECURSIVE SEQUENCES**

1) Which of these sequences cannot be modeled with an explicit formula?

A) 17, 23, 29, 35, ... B) 8, 11, 17, 29, ...

C) 3, 12, 48, 192, ...

- 2) A sequence is defined recursively by a(1) = 72 and a(n) = 5a(n-1) 11. How is the second term generated?
  - A) Multiply 71 by 5 and add 11.
  - B) Multiply 72 by 5 and subtract 11.
  - C) Multiply -1 by 5 and subtract 11.
- 3) Find the first four terms of the recursive sequence defined by:

$$a_1 = -2$$
;  $a_n = 3a_{n-1} - 4$ 

4) Which recursively defined sequence has a first term equal to 4 and a common ratio of 9?

A) f(1) = 9; f(n) = f(n-1) + 4 C) f(1) = 9; f(n) = 4f(n-1)

B) f(1) = 4; f(n) = f(n-1) + 9 D) f(1) = 4; f(n) = 9f(n-1)

5) If f(1) = 4 and  $f(n) = \frac{1}{4} f(n-1) + 8$ , then find the value of f(3).

### **RECURSIVE SEQUENCES**

- 1) Which of these sequences cannot be modeled with an explicit formula?
  - A) 17, 23, 29, 35, ...

**B) 8, 11, 17, 29, ...** C) 3, 12, 48, 192, ...

Arithmetic d= 6

geometric r = 4

Explicit: a(n) = 17 + 6(n - 1)

explicit:  $a(n) = 3(4)^{n-1}$ 

- 2) A sequence is defined recursively by a(1) = 72 and a(n) = 5a(n-1) 11. How is the second term generated?
  - A) Multiply 71 by 5 and add 11.
  - B) Multiply 72 by 5 and subtract 11.
  - C) Multiply -1 by 5 and subtract 11.
- 3) Find the first four terms of the recursive sequence defined by:

$$a_1 = -2$$
;  $a_n = 3a_{n-1} - 4$ 

$$a_2 = 3a_1 - 4 = 3(-2) - 4 = -10$$

$$a_3 = 3a_2 - 4 = 3(-10) - 4 = -34$$

$$a_4 = 3a_3 - 4 = 3(-34) - 4 = -106$$

-2, -10, -34, -106

4) Which recursively defined sequence has a first term equal to 4 and a common ratio of 9? f(1) = 4 and r = 9 which means you are multiplying by 9

A) 
$$f(1) = 9$$
;  $f(n) = f(n-1) + 4$  C)  $f(1) = 9$ ;  $f(n) = 4f(n-1)$ 

C) 
$$f(1) = 9$$
;  $f(n) = 4f(n-1)$ 

B) 
$$f(1) = 4$$
;  $f(n) = f(n-1) + 9$ 

B) 
$$f(1) = 4$$
;  $f(n) = f(n-1) + 9$  D)  $f(1) = 4$ ;  $f(n) = 9f(n-1)$ 

5) If f(1) = 4 and  $f(n) = \frac{1}{4} f(n-1) + 8$ , then find the value of f(3).

$$f(1) = 4$$
  $f(2) = \frac{1}{4}(4) + 8$   $f(3) = \frac{1}{4}(9) + 8$ 

$$f(3) = \frac{1}{4}(9) +$$

$$f(2) = 9$$

$$f(3) = 10.25$$

$$f(3) = 10.25$$