Study Guide-Quarter 1 Test

This is a summary of the material covered in all the units you have studied this marking quarter. To make sure that you are fully prepared for this test, you should **re-read your notes**, **re-watch all the flip videos** and **redo all the problems from your unit review sheets, unit tests and spiral sets.**

Unit 1 – The Real Number System

Real Numbers

Natural Numbers	Counting Numbers	{1,2,3,4,5,6,7…}
Whole Numbers	Counting Numbers & Zero	{0,1,2,3,4,5,6,7…}
Integers	Whole Numbers and their Opposites	{…-3,-2,-1,0,1,2,3…}
Rational Numbers	any number that can be written as a ratio of two integers $\frac{a}{b}, b \neq 0$ Includes all terminating and repeating decimals, cube roots of perfect cubes, square roots of perfect squares	27, $\frac{1}{4}$, $0.\overline{3}$, $5\frac{3}{5}$ -2, 0, $-\frac{2}{9}$, $\sqrt{16}$, $\sqrt[3]{27}$
Irrational Numbers	all non-terminating, non-repeating decimals, cube roots of non-perfect cubes, square roots of non-perfect squares	π, -$\sqrt{20}$, $\sqrt[3]{10}$

Simplifying Square Root Radicals

-Determine the largest perfect square that divides into the radicand evenly.

-Rewrite the radicand as a product.

-Rewrite the perfect square radical as a whole number.

Ex: **a)** $\sqrt{24}$ **b)** $\sqrt{32}$ **c)** $\sqrt{63}$

a)	b)	c)
$\sqrt{24}$	$\sqrt{32}$	$\sqrt{63}$
$\sqrt{4}\cdot\sqrt{6}$	$\sqrt{16}\cdot\sqrt{2}$	$\sqrt{9}\cdot\sqrt{7}$
$2\sqrt{6}$	$4\sqrt{2}$	$3\sqrt{7}$
$\mathbf{\sqrt{24} = 2\sqrt{6}}$	$\mathbf{\sqrt{32} = 4\sqrt{2}}$	$\mathbf{\sqrt{63} = 3\sqrt{7}}$

Calculator Check - *In order to make sure the original expression and the simplified expression are equivalent, type each expression into your calculator and press enter. You should see the same non-terminating, non-repeating decimal if you simplified correctly.*

Properties of Real Numbers

commutative	$a + b = b + a$ $ab = ba$	*Changing the order of addends or factors doesn't change the sum or product.*
associative	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$	*Regrouping addends or factors doesn't change the sum or product.*
identity	$a + 0 = a$ $(a)(1) = a$	*Any number or term added to zero or multiplied by 1 will always produce that number or term.*
inverse	$a + (-a) = 0$ $a\ (1/a) = 1$	*The sum of opposites (additive inverses) equals zero. The product of reciprocals (multiplicative inverses) equals one.*
distributive	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$	*A sum can be multiplied by a factor by multiplying each addend separately and then adding the products.*

Sums and Products of Rational and Irrational Numbers

-The **sum** or **product** of **two rational numbers** is always rational.

-The **sum** of a **rational number** and an **irrational number** is always irrational.

-The **product** of a **non-zero rational number** and an **irrational number** is always irrational.

-The **sum** or **product** of **two irrational numbers** may be rational or irrational.

Ex:

$\pi + \pi = 2\pi \leftarrow$ irrational sum $\quad \sqrt{3} \bullet \sqrt{5} = \sqrt{15} \leftarrow$ irrational product

$\pi + (-\pi) = 0 \leftarrow$ rational sum $\quad \sqrt{2} \bullet \sqrt{2} = \sqrt{4} = 2 \leftarrow$ rational product

Summation

- rational and rational = rational
- irrational and irrational = both
- rational and irrational = irrational

Exception: zero $\times$ irrational = rational

Unit 2 – Polynomial Expressions

Writing and Interpreting Expressions

- Translate verbal phrases into algebraic expressions by converting mathematical terminology into symbols

Ex: A nutritionist said that there are 60 calories in one brownie bite, 110 calories in an ounce of yogurt and 2 calories in one celery stick. The following expression represents the number of calories Mary consumed.

$$60b + 110y + 2c$$

a) What does **b** represent? ***The number of brownie bites Mary consumed***

b) What does **c** represent? ***The number of celery sticks Mary consumed***

c) What does **110y** represent? ***The number of calories Mary consumed by eating y ounces of yogurt***

d) What is the unit of measure associated with the expression? ***Calories***

e) If the expression was changed to **60b + 110y + 2y**, what conclusion can be drawn? ***The number of celery sticks consumed by Mary is equivalent to the number of ounces of yogurt she ate. The expression can also now be simplified to 60b + 112y.***

Evaluating Expressions and Formulas

- Substitute variables with numbers and simplify using the order of operations.

Ex: **a.** Evaluate $x^2 - y$ when $x = -5$ and $y = -6$

$x^2 - y$

$(-5)^2 - (-6)$

$25 + 6$

31

Put negative numbers in ( )

b. Find the number of Celsius degrees if the temperature is currently 70 degrees Fahrenheit.

$C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(70 - 32)$

$C = \frac{5}{9}(38)$

C = 21.1°

Operations with Polynomial Expressions

Polynomial Expressions are defined as a term or a sum/difference of terms in which each term contains variables raised to a whole number exponent.

Adding/Subtracting: Only "Like Terms" can be combine. Do not add exponents!

Distribute the minus sign

Ex: $5y^2 - y^2$

$4y^2$

Ex: $(7x^3 - 5x) + (-3x^2 + 6x) - (2x^2 - 3)$

$7x^3 - 5x - 3x^2 + 6x - 2x^2 + 3$

$7x^3 - 5x^2 + x + 3$

Multiplying Monomials & Polynomials: Multiply the coefficients and add the exponents of variables

Ex: $-5x^2(4x^3 - 2x)$

$-20x^5 + 10x^3$

When multiplying polynomials, distribute every term in the set of parentheses to every term in the other set of parentheses.

Ex: $(5x - 4)(x + 7)$

$5x(x + 7) - 4(x + 7)$

$5x^2 + 35x - 4x - 28$

$5x^2 + 31x - 28$

Ex: $(y - 2)(y^2 + 3y - 5)$

$\mathbf{y^3 + y^2 - 11y + 10}$

	y	-2
y^2	y^3	$-2y^2$
3y	$3y^2$	-6y
-5	-5y	10

Follow the order of operations (PEMDAS) when simplifying polynomial expressions.

Ex: Express $(x + 1)(2x - 4) - 3x + 5$ as a trinomial in standard form.

$(x + 1)(2x - 4) - 3x + 5$

$(2x^2 - 4x + 2x - 4) - 3x + 5$

$(2x^2 - 2x - 4) - 3x + 5$

$\mathbf{2x^2 - 2x - 4 - 3x + 5}$

$\mathbf{2x^2 - 5x + 1}$

Unit 3 – Equations

Solving Simple Equations

- *Simplify both sides of the equation using properties of real numbers*
- *Get the variable terms on one side of the equal sign and all constant terms on the other side using the properties of equality*
- *Use inverse operations (properties of equality) to solve for the variable*

Ex: $2(3x - 9) = 4x - 4 - 5x$ ***Always check solution*** $2(3(\mathbf{2})-9) = 4(\mathbf{2}) - 4 - 5(\mathbf{2})$

$6x - 18 = -x - 4$ $\qquad$ $2(6 - 9) = 8 - 4 - 10$

$7x = 14$ $\qquad$ $2(-3) = -6$

$x = 2$ $\qquad$ $-6 = -6$

Ex: Name the property of equality that was used for each result step.

$$\frac{2}{3}(9x + 6) = 9x - 2$$

$6x + 4 = 9x - 2$ [use the distributive property in order to simplify the left side]

$4 = 3x - 2$ [**subtraction property of equality**; subtracted 6x from each side]

$6 = 3x$ [**addition property of equality**; added 2 to each side]

$2 = x$ [**division property of equality**; divided both sides by 3]

Rational Equations

- *When two ratios are set equal to one another, they form a proportion.* ***Proportions*** *can be solved by* *<u>cross multiplying</u>*. ***Remember:*** *Put ( ) around expressions with more than 1 term.*

Ex: $\frac{x}{3} = \frac{5x+2}{18}$

$3(5x + 2) = 18(x)$

$15x + 6 = 18x$

$6 = 3x$

$\mathbf{2 = x}$

- *Solve rational equations with more than one term by multiplying every part of the equation by the **LCD** or by **creating a proportion**.*

Ex: Solve $\mathbf{\frac{x-2}{4}+\frac{1}{3}=\frac{7}{3}}$

Multiply by LCD $12\left(\frac{x-2}{4}+\frac{1}{3}\right) = 12\left(\frac{7}{3}\right)$

$$\cancel{12}^{3}\left(\frac{x-2}{\cancel{4}}\right)+\cancel{12}^{4}\left(\frac{1}{\cancel{3}}\right)=\cancel{12}^{4}\left(\frac{7}{\cancel{3}}\right)$$

$3(x-2) + 4(1) = 4(7)$

$3x - 6 + 4 = 28$

$3x - 2 = 28$

$\quad +2 \quad +2$

$\frac{3x}{3} = \frac{30}{3}$

$\boldsymbol{x = 10}$

Create a Proportion by "combining" fractions with a common denominator. Find equivalent fractions by multiplying by a **FOO** (*form of one*).

$$\frac{x-2}{4}+\frac{1}{3}=\frac{7}{3}$$

$$\mathbf{\frac{3}{3}}\bullet\frac{(x-2)}{4}+\frac{1}{3}\bullet\mathbf{\frac{4}{4}}=\frac{7}{3}$$

$$\frac{3x-6}{12}+\frac{4}{12}=\frac{7}{3}$$

$$\frac{3x-2}{12}=\frac{7}{3}$$

$3(3x-2) = 12(7)$

$9x - 6 = 84$

$9x = 90$

$\boldsymbol{x = 10}$

Solving Literal Equations (solving for another variable)

- *Solving literal equations means solve for a variable in terms of the other variables in the equation.*

Ex: *Solve for w in terms of A and l*

$A = lw$

$\frac{A}{l} = \frac{lw}{l}$

$\boldsymbol{\frac{A}{l} = w}$

Ex: Solve for *q* in terms of *a* and *m*:

$7q - a = m$

$\quad +a \quad +a$

$\frac{7q}{7} = \frac{m+a}{7}$

$\mathbf{q} = \boldsymbol{\frac{m+a}{7}}$

Solving Equations with No Solution or Infinite Solutions

- An **equation with infinite solutions** means that any number replacing the variable will make the statement true. The **solution set is all real numbers**.
- An **equation with no solution** means that no number when replaced with the variable will make the statement true.

This equation **has infinite solutions**.

Ex: $7x + 6 = 6 + 7x$

$-7x \qquad -7x$

$6 = 6$ **Constant = Same Constant**

x = all real numbers

This equation **has no solution**.

Ex: $8x - 9 = 8x + 2$

$-8x \qquad -8x$

$-9 \neq 2$ **Constant $\neq$ Different Constant**

Unit 4 – Applications with Equations

- ***When working with word problems….***
  -Read the problem carefully and make sense of the situation
  -Define all unknowns in terms of one variable (Let x = the unknown you know the least about)
  -Set up an equation relating the information in the problem to the unknowns
  -Solve and answer the question (label with appropriate units)
  -Tables can help organize information

Ex: Jane calculated that, on her day's intake of 2156 calories, four times as many calories were from carbohydrates than from protein, and twice as many calories were from fat than from protein. How many calories were from carbohydrates?

x: the number of calories from protein (**308**)
4x: the number of calories from carbohydrates (**4 • 308 = 1232**)
2x: the number of calories from fat (**2 • 308 = 616**)

$$x + 4x + 2x = 2156$$
$$\frac{7x}{7} = \frac{2156}{7}$$
$$x = 308$$

The number of calories from carbohydrates is 1232

Ex: Pam has two part time jobs. At one job, she works as a cashier and makes $8 per hour. At the second job, she works as a tutor and makes $12 per hour. One week she worked 30 hours and made $268. How many hours did she spend at each job?

x: number hours worked as a cashier

30 – x: number of hours worked as a tutor

$$\underline{8x} + \underline{12(30-x)} = 268$$

Total amount earned from both jobs

The amount of money earned as a cashier

The amount of money earned as a tutor

$$8x + 12(30 - x) = 268$$
$$8x + 360 - 12x = 268$$
$$-4x + 360 = 268$$
$$-4x = -92$$
$$x = 23$$

Check

$8(23 hrs) = $184 as a cashier

$12(7 hrs) = $84 as a tutor

$184 + $84 = $268

She worked at the cashier job for 23 hours and tutored for 7 hours.

It is important to remember that you should always ask yourself after solving a problem, "Does my answer make sense? Is it reasonable based on the situation?"

Consecutive Integers

- $x, x + 1, x + 2, x + 3$ …. *represent consecutive integers (including consecutive positive and negative integers)*

- $x, x + 2, x + 4, x + 6$ …. *represents consecutive even or odd integers*

Ex: Find three consecutive negative even integers such that the difference between the largest and smallest is 16 more than the second.

Let x = 1st negative even integer
Let $x + 2$ = 2nd negative even integer
Let $x + 4$ = 3rd negative even integer

the second

16 more than

$(x + 4) - (x) = (x + 2) + 16$

Largest difference smallest

$$(x + 4) - (x) = (x + 2) + 16$$
$$4 = x + 18$$
$$-14 = x$$

The integers are -14, -12 and -10

Coin, Stamp and Ticket

- Total Value of the Items($) = Value of one item × the number of Items

Ex: 10 coins made up of nickels and dimes are worth $0.65.

Let x = # of nickels
Let $10 - x$ = # of dimes

$5(x) + 10(10 - x) = 65$

value of one dime # of dimes

Type	Value	Qty	Total Value $
nickels	5	x	5x
dimes	10	10 – x	10(10 – x)

$$5x + 10(10 - x) = 65$$
$$5x + 100 - 10x = 65$$
$$-5x + 100 = 65$$
$$-5x = -35$$
$$x = 7$$

There are 7 nickels and 3 dimes

Check
7 nickels x $0.05 = $0.35
3 dimes x $0.10 = $0.30

Total: $0.35 + $0.30 = $0.65

Ex: Spotlight's "The Little Mermaid" sold 123 tickets for their Thursday afternoon production. The price of adult admission was $5 and the price of student admission was $3.50. If Spotlight earned $465 from ticket sales, how many of each type of ticket was sold?

x: number of student tickets
$123 - x$: number of adult tickets

Tickets	Value	Qty	Total Value $
Student Tickets	3.5	x	3.5x
Adult Tickets	5	123 – x	5(123 – x)

$$5(123 - x) + 3.5x = 465$$
$$615 - 5x + 3.5x = 465$$
$$615 - 1.5x = 465$$
$$-1.5x = -150$$
$$x = 100$$

100 student tickets were sold and 23 adult tickets were sold

Check
100 student tickets x $3.50 = $350
23 adult tickets x $5 = $115

Total: $350 + $115 = $465

Mixture

Ex: Caramels sell for \$1.20 per lb and taffies sell for \$1.90 per lb in the candy store. How many lbs of each type of candy were sold if an 18 lb mixture sold for \$25.80.

Candy	Value (\$/lb)	Quantity (lbs)	Total Value
Caramel	1.20	x	1.2x
Taffy	1.90	18 – x	1.9(18 – x)

$$1.2x + 1.9(18 - x) = 25.80$$
$$1.2x + 34.2 - 1.9x = 25.80$$
$$-0.7x + 34.2 = 25.80$$
$$-0.7x = -8.40$$
$$x = 12$$

$\underline{1.2x} + \underline{1.9(18 - x)} = 25.80$ ← total amount of money (\$) spent on the 18 lb mixture of caramels and taffies

total amount of money (\$) spent on caramels

total amount of money (\$) spent on taffies

12 lbs of caramels were sold and 6 lbs of taffy were sold

Ex: Twelve pounds of mixed nuts (brand A) contain 10% cashews. These nuts were combined with 8 pounds of another kind of mixed nuts (brand B). What percent of brand B was made up of cashews if the mixture of A and B was 30% cashews?

Mixed Nuts	% of cashews	total lbs of nuts	lbs of cashews
Brand A	0.10	12 lbs	0.10(12)
Brand B	x	8 lbs	x(8)

$$0.10(12) + x(8) = 0.30(20)$$
$$1.2 + 8x = 6$$
$$8x = 4.8$$
$$x = 0.6$$

$\underline{0.10(12)} + \underline{x(8)} = \underline{0.30(20)}$ ← total amount of lbs of cashews in the mixture (brands A and B)

lbs of cashews in brand A

lbs of cashews in brand B

60% of Brand B is cashews

Age

Ex: Jen is 10 years older than Phil. In 6 years, Phil will be ½ Jen's age. How old are they now?

Ages	Now	In 6 Years
Phil's age	x	x + 6
Jen's age	x + 10	(x + 10) + 6 → x + 16

$x + 6 = \frac{1}{2}(x + 16)$

will be ½

Phil's age in 6yrs

Jen's age in 6 yrs

$$x + 6 = \tfrac{1}{2}(x + 16)$$
$$x + 6 = 0.5x + 8$$
$$0.5x + 6 = 8$$
$$0.5x = 2$$
$$x = 4$$

Phil is 4 years old
Jen is 14 years old