Unit 5 Study Guide –*Applications with Equations (AKA: "Word Problems")*

I. GENERAL WORD PROBLEMS

KEY CONCEPT: When working with general word problems…

- Read the problem carefully and pull out important information (*list if necessary*)
- Define all unknowns in terms of one variable

  Ex: Jan ran twice as many miles on Tuesday than on Monday

  x = number of miles ran on Monday

  2x = number of miles ran on Tuesday
- Set up an equation relating all unknowns and solve
- Answer the question (find the unknowns)
- Check your solution(s) with the "words" of the problem

Remember: Always let x = the unknown you know the least about

When necessary, put ( ) around expressions with more than one term

Answer what is being asked

Examples:

1) A rectangle and square have the same perimeter. Each side of the square measures 10 cm. The length of the rectangle is three times its width. Find the dimensions of the rectangle.

   x: the width of the rectangle (5)

   3x: the length of the rectangle (3 • 5 = 15)

   Perimeter of the square = 4(10) = 40 cm

   Perimeter of the rectangle = 40 cm

   P = 2l + 2w

   40 = 2x + 2(3x)

   40 = 2x + 6x

   40 = 8x

   5 = x **The width is 5 cm and the length is 15 cm**

2) Jane calculated that, on her day's intake of 2156 calories, four times as many calories were from carbohydrates than from protein, and twice as many calories were from fat than from protein. How many calories were from carbohydrates?

   x: the number of calories from protein (308)

   4x: the number of calories from carbohydrates (4 • 308 = 1232)

   2x: the number of calories from fat (2 • 308 = 616)

   x + 4x + 2x = 2156 [protein calories + carbohydrate calories + fat calories = total calories]

   7x = 2156

   x = 308 **The number of calories from carbohydrates is 1232**

II. CONSECUTIVE INTEGER WORD PROBLEMS

KEY CONCEPT:

- Variables used for finding **consecutive integers** (x, $x + 1$, $x + 2$, etc…)
  Consecutive integers show a difference of 1
- Variables used for finding either **even or odd consecutive integers** (x, $x + 2$, $x + 4$, etc…)
  Consecutive even or odd integers show a difference of two

Remember: When necessary, put ( ) around expressions with more than one term
Answer what is being asked

Examples:

1) Find three consecutive integers such that 5 times the second is 22 more than twice the sum of the smallest and largest

$$\overset{\text{5 times}}{5}(\underset{\text{the 2nd}}{x + 1}) \overset{\text{is}}{=} \underset{\text{22 more than}}{22} + \overset{\text{twice}}{2}(\underset{\text{the sum of the smallest and largest}}{x + x + 2})$$

x: 1^{st} consecutive integer
$x + 1$: 2^{nd} consecutive integer
$x + 2$: 3^{rd} consecutive integer

$$5(x + 1) = 22 + 2(x + x + 2)$$
$$5x + 5 = 22 + 2x + 2x + 4$$
$$5x + 5 = 26 + 4x$$
$$5x = 21 + 4x$$
$$x = 21$$

The integers are 21, 22 and 23

2) Find three consecutive odd integers whose sum is -81.

x: 1^{st} consecutive odd integer (-29)
$x + 2$: 2^{nd} consecutive odd integer (-29 + 2 = -27)
$x + 4$: 3^{rd} consecutive odd integer (-29 + 4 = -25)

$$x + (x + 2) + (x + 4) = -81 \qquad [1^{st} \text{ odd int.} + 2^{nd} \text{ odd int.} + 3^{rd} \text{ odd int.} = \text{sum}]$$
$$3x + 6 = -81$$
$$3x = -87$$
$$x = -29$$

The integers are -29, -27 and -25

III. MONEY WORD PROBLEMS (COIN, STAMP & TICKET)

KEY CONCEPT:

- Value of an item = the value of each item (*worth*) × the number of items (*how many*)
  Ex: the value of 10 stamps worth 2 cents each = 10(2) = 20 cents

Remember: You can work in pennies by multiplying both sides of the equation by 100
Set up a table to find the total value of an item
Use ( ) when an expression has more than 1 term

Examples:

1) A person has 23 coins made up dimes and quarters worth $3.35. How many coins of each type are there?

Coins	Value	Quantity	Total Value
Dimes	.10	x	.10x
Quarters	.25	23 - x	.25(23 – x)

x: the number of dimes
23 – x: the number of quarters

$$.10x + .25(23 - x) = 3.35$$
$$10x + 25(23 - x) = 335$$
$$10x + 575 - 25x = 335$$
$$-15x + 575 = 335$$
$$-15x = -240$$
$$x = 16$$

$$\underbrace{10x}_{\text{total amount of money (\$) from dimes}} + \underbrace{25(23 - x)}_{\text{total amount of money (\$) from quarters}} = \underbrace{335}_{\text{total amount of money (\$) from dimes and quarters}}$$

There are 16 dimes and 7 quarters

2) Spotlight's production of The Little Mermaid sold 123 tickets for the Thursday afternoon showing. The price of adult admission was $5 and the price of student admission was $3.50. If Spotlight earned $465 from ticket sales, how many of each type of ticket was sold?

Tickets	Value	Quantity	Total Value
Student Tickets	3.50	x	3.50x
Adult Tickets	5	123 – x	5(123 – x)

x: number of student tickets
123 – x: number of adult tickets

$$\underbrace{5(123 - x)}_{\text{total amount of money (\$) earned from adult tickets}} + \underbrace{3.5x}_{\text{total amount of money (\$) earned from student tickets}} = \underbrace{465}_{\text{total amount of money (\$) earned from student tickets and adult tickets}}$$

$$5(123 - x) + 3.5x = 465$$
$$615 - 5x + 3.5x = 465$$
$$615 - 1.5x = 465$$
$$-1.5x = -150$$
$$x = 100$$

100 student tickets were sold and 23 adult tickets were sold

IV. MIXTURE WORD PROBLEMS

KEY CONCEPT: Multiply $\rightarrow$(% or the value per unit) $\times$ (quantity)

Remember: "Money" is not always part of the problem
Set up a table to organize the information

Examples:

1) Caramels sell for \$1.20 per lb and taffies sell for \$1.90 per lb in the candy store. How many lbs of each type of candy were sold if a 18 lb mixture sold for \$25.80.

Candy	Value (\$/lb)	Quantity (lbs)	Total Value
Caramel	1.20	x	1.2x
Taffy	1.90	18 – x	1.9(18 – x)

$$1.2x + 1.9(18 - x) = 25.80$$
$$1.2x + 34.2 - 1.9x = 25.80$$
$$-0.7x + 34.2 = 25.80$$
$$-0.7x = -8.40$$
$$x = 12$$

$$1.2x + 1.9(18 - x) = 25.80$$

total amount of money (\$) spent on the 18 lb mixture of caramels and taffies

total amount of money (\$) spent on caramels

total amount of money (\$) spent on taffies

12 lbs of caramels were sold and 6 lbs of taffy was sold

2) Twelve pounds of mixed nuts (brand A) contains 10% cashews. These nuts were mixed with 8 pounds of another kind of mixed nuts (brand B). What percent of brand B was made up of cashews if the mixture of A and B was 30% cashews?

Mixed Nuts	% of cashews	total lbs of nuts	lbs of cashews
Brand A	.10	12 lbs	.10(12)
Brand B	x	8 lbs	x(8)

$$.10(12) + x(8) = .30(20)$$
$$1.2 + 8x = 6$$
$$8x = 4.8$$
$$x = .6$$

$$.10(12) + x(8) = .30(20)$$

total amount of lbs of cashews in the mixture (brands A and B).

lbs of cashews in brand A

lbs of cashews in brand B

60% of the mixture is cashews

V. AGE WORD PROBLEMS

KEY CONCEPT:

- Define the ages now (present ages)
- Define the ages in the past and/or future
- Set up an equation that represents the relationship described between the ages

Remember: Set up a table to organize the information

Example: Jen is 10 years older than Phil. In 6 years, Phil will be ½ Jen's age. How old are they now?

Ages	Now	In 6 Years
Phil's age	x	x + 6
Jen's age	x + 10	(x + 10) + 6 → x + 16

$$\underbrace{x + 6}_{\text{Phil's age in 6yrs}} \underbrace{=\ \tfrac{1}{2}}_{\text{will be ½}} \underbrace{(x + 16)}_{\text{Jen's age in 6 yrs}}$$

$$x + 6 = \tfrac{1}{2}(x + 16)$$
$$x + 6 = 0.5x + 8$$
$$0.5x + 6 = 8$$
$$0.5x = 2$$
$$x = 4$$

Phil is 4 years old and Jen is 14 years old.

VI. WORK WORD PROBLEMS

KEY CONCEPT:

- $$\frac{1}{\text{time it takes 1st person/object to complete job}} + \frac{1}{\text{time it takes 2nd person/object to complete job}} = \frac{1}{\text{total time it takes to complete job together}}$$

Remember: Define your unknown (x = ?)
In order to solve the rational equation, create a proportion

Example: Jerry can trim the bushes in 5 hours but Alan can do it in 3 hours. How long will it take if both boys to do the job together?

x: the number of hours it takes to do the job together

$$\underbrace{\frac{1}{5}}_{\text{the rate at which Jerry can complete the job}} + \underbrace{\frac{1}{3}}_{\text{the rate at which Alan can complete the job}} = \underbrace{\frac{1}{x}}_{\text{the rate at which the boys can complete the job together}}$$

$$\frac{1}{5} + \frac{1}{3} = \frac{1}{x}$$
$$\frac{3}{15} + \frac{5}{15} = \frac{1}{x}$$
$$\frac{8}{15} = \frac{1}{x}$$

$$8(x) = 15(1)$$
$$8x = 15$$
$$x = \frac{15}{8} = 1\frac{7}{8}$$

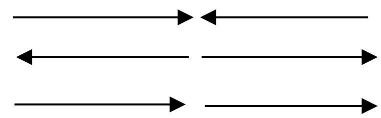
It will take $1\frac{7}{8}$ hours to complete trim the bushes

VII. Distance, Rate, Time

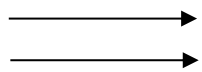
KEY CONCEPT: **D = Rate × Time**

Remember:

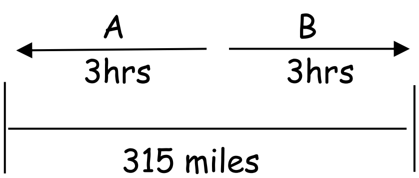
Distance of A + Distance of B = Total Distance of A and B



Distance A = Distance B



Example: Two trains leave at the same station in opposite directions. One train travels 15mph faster than the other. After 3 hrs, the trains are 315 miles apart. Find the rate of each train.



Let x = rate of A **45 mph**

Let x + 15 = rate of B **60 mph**

A's Distance + B's Distance = Total Distance (A&B)

RT + RT = D

$$(x)(3) + (x + 15)(3) = 315$$
$$3x + 3x + 45 = 315$$
$$6x + 45 = 315$$
$$6x = 270$$
$$x = 45$$