# Algebra 1 Midterm Study Guide

## 1)The Real Number System and Properties

### A. Real Numbers

| | | |
|---|---|---|
| Natural Numbers | counting numbers | {1,2,3,4,5,6,7…} |
| Whole Numbers | counting numbers & zero | {0,1,2,3,4,5,6,7…} |
| Integers | whole numbers and their opposites | {…-3,-2,-1,0,1,2,3…} |
| Rational Numbers | any number that can be written as a ratio of two integers $\frac{a}{b}, b \neq 0$ Includes all terminating and repeating decimals, cube roots of perfect cubes, square roots of perfect squares | 27, $\frac{1}{4}$, $0.\bar{3}$, $5\frac{3}{5}$ -2, 0, $-\frac{2}{9}$, $\sqrt{16}$, $\sqrt[3]{27}$ |
| Irrational Numbers | all non-terminating, non-repeating decimals, cube roots of non-perfect cubes, square roots of non-perfect squares | $\pi$, -$\sqrt{20}$, $\sqrt[3]{10}$ |

### B. Properties of Real Numbers

| | | |
|---|---|---|
| commutative | a + b = b + a<br>ab = ba | *Changing the order of addends or factors doesn't change the sum or product.* |
| associative | (a + b)+ c = a + (b + c)<br>(ab)c = a(bc) | *Regrouping addends or factors doesn't change the sum or product.* |
| identity | a + 0 = a<br>(a)(1) = a | *Any number or term added to zero or multiplied by 1 will always produce that number or term.* |
| inverse | a + (-a) = 0<br>a (1/a) = 1 | *The sum of opposites (additive inverses) equals zero. The product of reciprocals (multiplicative inverses) equals one.* |
| distributive | a(b + c) = ab + ac<br>a(b – c) = ab – ac | *A sum can be multiplied by a factor by multiplying each addend separately and then adding the products.* |
| zero product | (a)(0) = 0 | *Any term or number multiplied by zero is always zero.* |

## C. Sums and Products of Rational and Irrational Numbers

-The **sum** or **product** of two **rational** numbers is **always rational**.

-The **sum** of a **rational** number and an **irrational** number is **always irrational**.

$2 + \pi = 2 + \pi$ (irrational)

$2 - \pi = 2 - \pi$ (irrational)

-The **product** of a **non-zero rational** number and an **irrational** number is **always irrational**.

$2 \times \pi = 2\pi$ (irrational)

-The **sum** or **product** of **two irrational** numbers may be **rational** or **irrational**.

| Irrational | Operation | Irrational | Result | Rational/Irrational |
|---|---|---|---|---|
| $\pi$ | + | $\pi$ | $2\pi$ | Irrational |
| $-\sqrt{6}$ | + | $\sqrt{6}$ | 0 | Rational |
| $\sqrt{2}$ | x | $\sqrt{6}$ | $\sqrt{12}$ | Irrational |
| $\sqrt{2}$ | x | $\sqrt{8}$ | $\sqrt{16} = 4$ | Rational |

## D. Simplifying Square Root Radicals

-Determine the largest perfect square that divides into the radicand evenly.

-Rewrite the radicand as a product.

-Rewrite the perfect square radical as a whole number.

**Ex:**

a) $\sqrt{24}$
$\sqrt{4} \cdot \sqrt{6}$
$2\sqrt{6}$

$\mathbf{\sqrt{24} = 2\sqrt{6}}$

b) $\sqrt{32}$
$\sqrt{16} \cdot \sqrt{2}$
$4\sqrt{2}$

$\mathbf{\sqrt{32} = 4\sqrt{2}}$

c) $\sqrt{63}$
$\sqrt{9} \cdot \sqrt{7}$
$3\sqrt{7}$

$\mathbf{\sqrt{63} = 3\sqrt{7}}$

***Calculator Check***
*In order to make sure the original irrational term and the simplified irrational term are equivalent, type each into your calculator and press enter. You should see the same non-terminating, non-repeating decimal if you simplified correctly.*

# 2)Polynomial Expressions

## A. Operations with Polynomials

- Add and subtract polynomials by combining like terms (*remember to distribute the minus sign when subtracting*).

**Ex:** $(3x - 2) + (5x^2 + x) - (7x + 4)$

$3x - 2 + 5x^2 + x - 7x - 4$

$\mathbf{3x - 2 + 5x^2 + x - 7x - 4}$

$\mathbf{5x^2 - 3x - 6}$ ← *All final answers should be written in standard form.*

- Multiply polynomials by multiplying coefficients and adding exponents of like variables (*use the distributive property when necessary*)

**Ex: a.** $3x(2x^2 - 6x + 1)$
$\mathbf{6x^3 - 18x^2 + 3x}$

**b.** $(3x - 2)(x + 4)$
$3x^2 + 12x - 2x - 8$
$\mathbf{3x^2 + 10x - 8}$

**c.** $(x + 5)(x^2 - 4x + 9)$

| | $x^2$ | $-4x$ | $+9$ |
|---|---|---|---|
| $x$ | $\mathbf{x^3}$ | $\mathbf{-4x^2}$ | $\mathbf{9x}$ |
| $5$ | $\mathbf{5x^2}$ | $\mathbf{-20x}$ | $\mathbf{45}$ |

$\mathbf{x^3 + x^2 - 11x + 45}$

***Every factor in the first set of ( ) must be distributed to every factor in the second set of ( )***

- Divide polynomials by monomials by dividing each term of the polynomial by the monomial. Divide coefficients and subtract exponents of like variables.

**Ex:**

$$\frac{6x^2 - 4x + 2}{2}$$

$$\frac{6x^2}{2} - \frac{4x}{2} + \frac{2}{2}$$

$\mathbf{3x^2 - 2x + 1}$

***Check*** Distribute

$2(3x^2 - 2x + 1) = 6x^2 - 4x + 2$

## B. Writing and Interpreting Expressions

- Translate verbal phrases into algebraic expressions by converting mathematical terminology into symbols

**Ex:** A nutritionist said that there are 60 calories in one brownie bite, 110 calories in an ounce of yogurt and 2 calories in one celery stick. The following expression represents the number of calories Mary consumed.

$$60b + 110y + 2c$$

a) What does **b** represent? ***The number of brownie bites Mary consumed***

b) What does **c** represent? ***The number of celery sticks Mary consumed***

c) What does **110y** represent? ***The number of calories Mary consumed by eating y ounces of yogurt***

d) What is the unit of measure associated with the expression? ***Calories***

e) If the expression was changed to 60b + 110y + 2y, what conclusion can be drawn? ***The number of celery sticks consumed by Mary is equivalent to the number of ounces of yogurt she ate. The expression can also now be simplified to 60b + 112y.***

## C. Evaluating Expressions and Formulas

- Substitute variables with numbers and simplify using the order of operations.

**Ex: a.** Evaluate $x^2 - y$ when x = -5 and y = -6

$x^2 - y$

$(-5)^2 - (-6)$

$25 + 6$

**31**

***Put negative numbers in ( )***

**b.** Find the number of Celsius degrees if the temperature is currently 70 degrees Fahrenheit.

$C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(70 - 32)$

$C = \frac{5}{9}(38)$

**C = 21.1°**

# 3) Equations

## A. Solving Simple Equations

- *Simplify both sides of the equation using properties of real numbers*
- *Get the variable terms on one side of the equal sign and all number terms on the other side using the properties of equality*
- *Use inverse operations (properties of equality) to solve for the variable*

**Ex:** $2(3x - 9) = 4x - 4 - 5x$

$6x - 18 = -x - 4$

$7x = 14$

$x = 2$

***Always check solution*** $2(3(2)-9) = 4(2) - 4 - 5(2)$

$2(6 - 9) = 8 - 4 - 10$

$2(-3) = -6$

$-6 = -6$

**Ex:** Name the property of equality that was used for each result step.

$\frac{2}{3}(9x + 6) = 9x - 2$

$6x + 4 = 9x - 2$ [use the distributive property in order to simplify the left side]

$4 = 3x - 2$ [**subtraction property of equality**; subtracted 6x from each side]

$6 = 3x$ [**addition property of equality**; added 2 to each side]

$2 = x$ [**division property of equality**; divided both sides by 3]

## B. Solving Rational Equations

- *When two ratios are set equal to one another, they form a proportion.* ***Proportions*** *can be solved by* <u>*cross multiplying*</u>. ***Remember:*** *Put ( ) around expressions with more than 1 term.*

**Ex:** $\frac{x}{3} = \frac{5x+2}{18}$

$3(5x + 2) = 18(x)$

$15x + 6 = 18x$

$6 = 3x$

$\mathbf{2 = x}$

- *Solve rational equations with more than one term by multiplying every part of the equation by the* ***LCD*** *or by* ***creating a proportion****.*

**Ex:** Solve $\frac{x-2}{4}+\frac{1}{3}=\frac{7}{3}$

**Multiply by LCD** $12(\frac{x-2}{4}+\frac{1}{3}) = 12(\frac{7}{3})$

$$\not{1}\not{2}^{3}\left(\frac{x-2}{\not{4}}\right)+\not{1}\not{2}^{4}\left(\frac{1}{\not{3}}\right)=\not{1}\not{2}^{4}\left(\frac{7}{\not{3}}\right)$$

$$3(x-2)+4(1)=4(7)$$
$$3x-6+4=28$$
$$3x-2=28$$
$$\quad +2 \quad +2$$
$$\frac{3x}{3}=\frac{30}{3}$$
$$\mathbf{x = 10}$$

**Create a Proportion** by "combining" fractions with a common denominator. Find equivalent fractions by multiplying by a **FOO** (*form of one*).

$$\frac{x-2}{4}+\frac{1}{3}=\frac{7}{3}$$
$$\frac{3}{3}\bullet\frac{(x-2)}{4}+\frac{1}{3}\bullet\frac{4}{4}=\frac{7}{3}$$
$$\frac{3x-6}{12}+\frac{4}{12}=\frac{7}{3}$$
$$\frac{3x-2}{12}=\frac{7}{3}$$

$$3(3x-2)=12(7)$$
$$9x-6=84$$
$$9x=90$$
$$\mathbf{x = 10}$$

## C. Solving Literal Equations (*solving for another variable*)

- *Solving literal equations means solve for a variable in terms of the other variables in the equation.*

**Ex:** *Solve for* ***w*** *in terms of* ***A*** *and* ***l***

$$A = lw$$
$$\frac{A}{l}=\frac{lw}{l}$$
$$\frac{A}{l}=w$$

**Ex:** Solve for ***q*** in terms of ***a*** and ***m***:

$$7q - a = m$$
$$\quad +a \quad +a$$
$$\frac{7q}{7}=\frac{m+a}{7}$$
$$q=\frac{m+a}{7}$$

## D. Solving Equations with No Solution or Infinite Solutions

An **equation with infinite solutions** means that any number replaced by the variable will make the statement true.

An **equation with no solution** means that there is no number that, when replaced with the variable, will make the statement true.

This equation **has infinite solutions**.

Ex: $7x + 6 = 6 + 7x$
$\quad -7x \qquad -7x$
$6 = 6$ **Constant = Same Constant**

**x = all real numbers**

This equation **has no solution**.

Ex: $8x - 9 = 8x + 2$
$\quad -8x \qquad -8x$
$-9 \neq 2$ **Constant ≠ Different Constant**

## E. Using Equations to Solve Word Problems

- ***When working with word problems….***
  ***-Read the problem carefully and recognize important information (list if necessary)***
  ***-Define all unknowns in the same variable (Let x = the unknown you know the least about)***
  ***-Set up an equation relating all unknowns***
  ***-Solve the equation***
  ***-Answer the question (label with appropriate units)***
  ***-Check answer: is it reasonable?***
  ***-Tables can help organize information***

**Ex:** Jane calculated that, on her day's intake of 2156 calories, four times as many calories were from carbohydrates than from protein, and twice as many calories were from fat than from protein. How many calories were from carbohydrates?

$x$: the number of calories from protein (**308**)
$4x$: the number of calories from carbohydrates ($4 \bullet 308 = 1232$)
$2x$: the number of calories from fat ($2 \bullet 308 = 616$)

$$x + 4x + 2x = 2156$$
$$\frac{7x}{7} = \frac{2156}{7}$$
$$x = 308$$

**The number of calories from carbohydrates is 1232**

**Ex:** Pam has two part time jobs. At one job, she works as a cashier and makes \$8 per hour. At the second job, she works as a tutor and makes \$12 per hour. One week she worked 30 hours and made \$268. How many hours did she spend at each job?

$x$: number hours worked as a cashier
$30 - x$: number of hours worked as a tutor

$$8x + 12(30 - x) = 268$$

**Total amount earned from both jobs**

**The amount of money earned as a cashier**

**The amount of money earned as a tutor**

$$8x + 12(30 - x) = 268$$
$$8x + 360 - 12x = 268$$
$$-4x + 360 = 268$$
$$-4x = -92$$
$$x = 23$$

***Check***
\$8(23 hrs) = \$184 as a cashier
\$12(7 hrs) = \$84 as a tutor

\$184 + \$84 = \$268

**She worked at the cashier job for 23 hours and tutored for 7 hours.**

***It is important to remember that you always ask yourself after solving a problem, "Does my answer make sense? Is it reasonable based on the situation?"***

## Consecutive Integers

- x, x + 1, x + 2, x + 3 …. *represent consecutive integers (including consecutive positive and negative integers)* → Pattern: + 1

- x, x + 2, x + 4, x + 6 …. *represents consecutive even or odd integers* → Pattern: +2

**Ex:** Find three consecutive negative even integers such that the difference between the largest and smallest is 16 more than the second.

Let x = 1st negative even integer
Let x + 2 = 2nd negative even integer
Let x + 4 = 3rd negative even integer

**the second**
**16 more than**

**x + 4 – x = x + 2 + 16**

**Largest** **difference** **smallest**

$x + 4 - x = 16 + x + 2$
$4 = 18 + x$
$-14 = x$

**The integers are -14, -12 and -10**

## Coin, Stamp and Ticket

- Total Value of the Items($) = Worth of the Items × # of Items

**Ex:** 10 coins made up of nickels and dimes are worth $0.65.

| Type | Value | Qty | Total Value ($) |
|---|---|---|---|
| nickels | 5 | x | 5x |
| dimes | 10 | 10 – x | 10(10 – x ) |

Let x = # of nickels
Let 10 – x = # of dimes

$5(x) + 10(10 - x) = 65$

Worth of coin     # of coins

$5x + 10(10 - x) = 65$
$5x + 100 - 10x = 65$
$-5x + 100 = 65$
$-5x = -35$
$x = 7$

**There are 7 nickels and 3 dimes**

***Check***
*7 nickels x $0.05 = $0.35*
*3 dimes x $0.10 = $0.30*

*Total: $0.35 + $0.30 = $0.65*

**Ex:** Spotlight's "The Little Mermaid" sold 123 tickets for their Thursday afternoon production. The price of adult admission was $5 and the price of student admission was $3.50. If Spotlight earned $465 from ticket sales, how many of each type of ticket was sold?

x: number of student tickets
123 – x: number of adult tickets

| Tickets | Value | Qty | Total Value ($) |
|---|---|---|---|
| Student Tickets | 3.5 | x | 3.5x |
| Adult Tickets | 5 | 123 – x | 5(123 – x) |

$5(123 - x) + 3.5x = 465$
$615 - 5x + 3.5x = 465$
$615 - 1.5x = 465$
$-1.5x = -150$
$x = 100$

**100 student tickets were sold and 23 adult tickets were sold**

***Check***
*100 student tickets x $3.50 = $350*
*23 adult tickets x $5 = $115*

*Total: $350 + $115 = $465*

## Mixture

**Ex:** Caramels sell for $1.20 per lb and taffies sell for $1.90 per lb in the candy store. How many lbs of each type of candy were sold if an 18 lb mixture sold for $25.80?

| Candy | Value ($/lb) | Quantity (lbs) | Total Value ($) |
|---|---|---|---|
| Caramel | 1.20 | x | 1.2x |
| Taffy | 1.90 | 18 – x | 1.9(18 – x) |

$$1.2x + 1.9(18 - x) = 25.80$$
$$1.2x + 34.2 - 1.9x = 25.80$$
$$-0.7x + 34.2 = 25.80$$
$$-0.7x = -8.40$$
$$x = 12$$

$$1.2x + 1.9(18 - x) = 25.80$$

1.2x: total amount of money ($) spent on caramels

1.9(18 – x): total amount of money ($) spent on taffies

25.80: total amount of money ($) spent on the 18 lb mixture of caramels and taffies

**12 lbs of caramels were sold and 6 lbs of taffy were sold**

**Ex:** Twelve pounds of mixed nuts (brand A) contain 10% cashews. These nuts were combined with 8 pounds of another kind of mixed nuts (brand B). What percent of brand B was made up of cashews if the mixture of A and B was 30% cashews?

| Mixed Nuts | % of cashews | total lbs of nuts | lbs of cashews |
|---|---|---|---|
| Brand A | .10 | 12 lbs | .10(12) |
| Brand B | x | 8 lbs | x(8) |

$$.10(12) + x(8) = .30(20)$$
$$1.2 + 8x = 6$$
$$8x = 4.8$$
$$x = .6$$

$$.10(12) + x(8) = .30(20)$$

.10(12): lbs of cashews in brand A

x(8): lbs of cashews in brand B

.30(20): total amount of lbs of cashews in the mixture (brands A and B).

**60% of Brand B is cashews**

## Age

**Ex:** Jen is 10 years older than Phil. In 6 years, Phil will be ½ Jen's age. How old are they now?

| Ages | Now | In 6 Years |
|---|---|---|
| Phil's age | x | x + 6 |
| Jen's age | x + 10 | (x + 10) + 6 → x + 16 |

$$x + 6 = \frac{1}{2}(x + 16)$$

x + 6: Phil's age in 6yrs

= ½: will be ½

(x + 16): Jen's age in 6 yrs

$$x + 6 = \frac{1}{2}(x + 16)$$
$$x + 6 = 0.5x + 8$$
$$0.5x + 6 = 8$$
$$0.5x = 2$$
$$x = 4$$

**Phil is 4 years old and Jen is 14 years old**

# 4)Inequalities

## A. Solving Simple Inequalities

- An **inequality** is a mathematical statement containing one of the symbols $<$ , $>$ , $\leq$ , $\geq$ , $\neq$ to indicate the relationship between two quantities.

- The solution to an inequality is any number that makes the inequality true. We solve simple inequalities using the same algebraic procedures that we use for equations. However, when we **multiply or divide both sides of an inequality by a negative number, we must flip the inequality symbol**.

- When graphing the solution set to an inequality on a number line, remember the following rules.

  $<$ or $>$ uses an open circle (*do not include*) ***and*** $\leq$ or $\geq$ uses a closed circle (*include*)

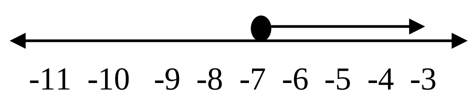
**Ex:**

$-2x - 3 \leq 11$

$-2x \leq 14$

$\mathbf{x \geq -7}$ also written as **$[-7, \infty)$**

Any real number greater than or equal to -7 will make this inequality true.




## B. Solving Word Problems with Simple Inequalities

- Read the problem carefully and identify the unknown (***make sure to assign a variable to represent the unknown***).

- Write an inequality that establishes a relationship between the unknown and the quantities in the problem. Solve and answer the question.

**Ex:** Dan has $70 to spend on CDs. Each CD costs $12 including tax. How many CDs can Dan buy?

x = the number of CDs Dan can buy.

$12x \leq 70$ ← *Dan can only spend $70 or less on CDs*

$x \leq 5\frac{5}{6}$ ← *Dan cannot buy a fraction of a CD*

**Dan can buy at most 5 CDs.**

**Ex:** Suppose you are signing up for a cable television service plan. Plan *A* costs $11 per month plus $7 for each premium channel. Plan *B* costs $27 per month plus $3 for each premium channel. For what number of premium channels will Plan *B* be more cost effective?

x = the number of premium channels.

*Plan B cheaper than (<) Plan A*

Plan *B* < Plan *A*

$27 + 3x < 11 + 7x$

$-4x < -16$

$x > 4$

**Plan B is more cost effective if 5 or more premium channels are ordered.**

## C. <u>Compound Inequalities</u>

- A **compound inequality** is a sentence with two inequality statements joined either by the word "**or**" or by the word "**and**."
- "**And**" indicates that both statements of the compound sentence are true at the same time. It is the overlap or intersection of the solution sets for the individual statements.
- "**Or**" indicates that, as long as either statement is true, the entire compound sentence is true. It is the combination or union of the solution sets for the individual statements.

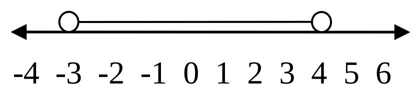
**Ex:** Solve for x

$3x + 2 < 14$ and $2x - 5 > -11$ $\rightarrow$ *Solve each inequality separately. Since the joining word is "and", graph the overlap or intersection of the result.*

$3x + 2 < 14$ and $2x - 5 > -11$

$3x < 12$ and $2x > -6$

$x < 4$ and $x > -3$




**(-3, 4)** $\leftarrow$ interval notation

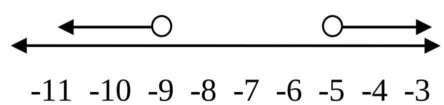
**Ex:** Solve for x

$2x + 7 < -11$ or $-3x - 2 < 13$ $\rightarrow$ *Solve each inequality separately. Since the joining word is "or", combine the answers; graph both solution sets.*

$2x + 7 < -11$ or $-3x - 2 < 13$

$2x < -18$ or $-3x < 15$

$x < -9$ or $x > -5$




$(-\infty,-9) \vee (-5,\infty)$ $\leftarrow$ interval notation

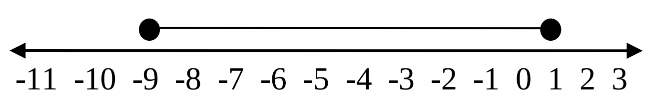
**Ex:** Solve for x

$-12 \leq 2x + 6 \leq 8$ $\leftarrow$ *Since this inequality has no connecting word written, it is understood to be "and". It is translated into the following compound sentence.*

$-12 \leq 2x + 6$ and $2x + 6 \leq 8$

$-18 \leq 2x$ and $2x \leq 2$

$-9 \leq x$ and $x \leq 1$




**[-9, 1]** $\leftarrow$ interval notation

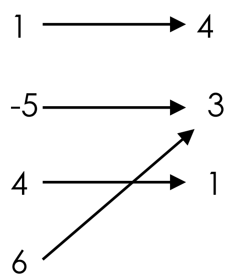
# 5) Linear Functions

## A. Functions

- **Relation:** a set of ordered pairs ($x$, $y$)
- **Function:** a relation in which each *x-value* is paired with exactly one *y-value*.

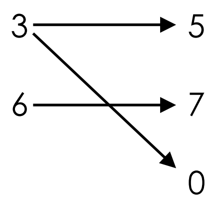
**Ex:** This relation represents a **function** because each input is paired with one output.

{(1,4), (-5,3), (4,1), (6,3)}




This relation is **not a function** because the input 3 has been assigned to more than one output.

{(3,5) (6,7) (3,0)}



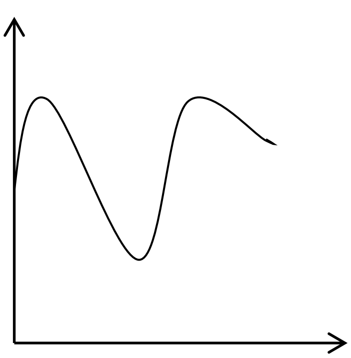

- Associated with each function is a ***domain*** and ***range***.

The **domain** of a function is the set of all x-values of its ordered pairs (input values or independent variables).

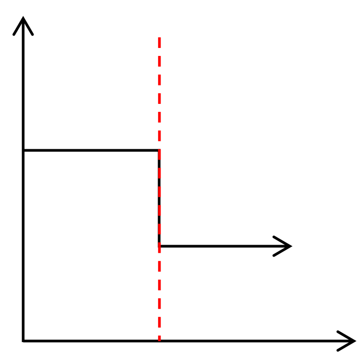
The **range** of a function is the set of all y-values of its ordered pairs (output values or dependent variables).

A test called the **vertical line test** can be used to determine if a graph is a function. In this test, a vertical line is moved across the graph from left to right. If the graph is intersected by the vertical line in more than one place at a time, the graph is not a function.

**Ex:**



**Function**



**Not a Function**

B. **Graphing Linear Functions**

- A linear function is a picture of infinite points that create a non-vertical line.
- There are three ways to graph a line.

-Creating a Table of x and y Values
-Intercept Method (finding the x and y-intercepts → (x,0) and (0,y))
-Slope-Intercept Method "y = *m*x + *b*",
where ***m*** represents the slope and ***b*** represents the y-intercept.

**Ex:** Graph: 2y + x = -6

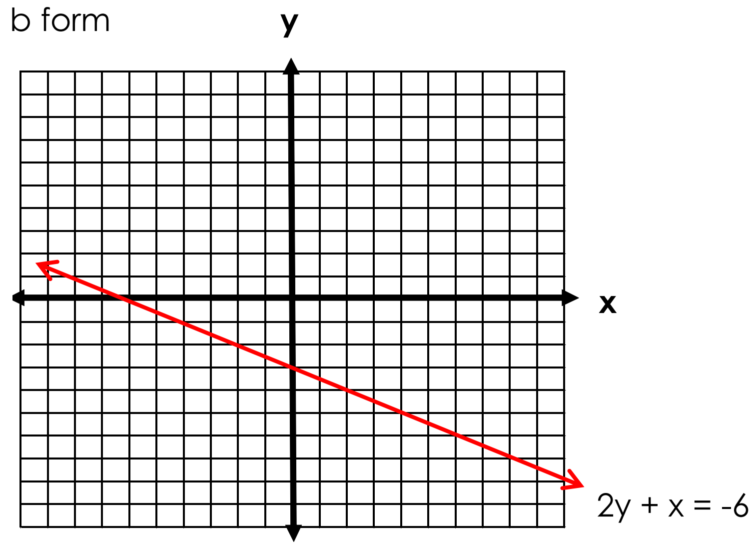
$\frac{2y}{2} = \frac{-x}{2} - \frac{6}{2}$ ← put in y = mx = b form

$y = -\frac{1}{2}x - 3$

| x | y |
|---|---|
| -4 | -1 |
| -2 | -2 |
| 0 | -3 |
| 2 | -4 |
| 4 | -5 |

or $m = \frac{-1\downarrow}{2\rightarrow} \text{ or } \frac{\uparrow}{\leftarrow}$

b = - 3 (0,-3)




Graph: $\frac{2}{3}x - \frac{1}{2}y = -2$

x-int: (X,0) Let y = 0

$\frac{2}{3}x - \frac{1}{2}(0) = -2$

$\frac{2}{3}x = -2$

$x = -3$

x-int: -3 (-3,0)

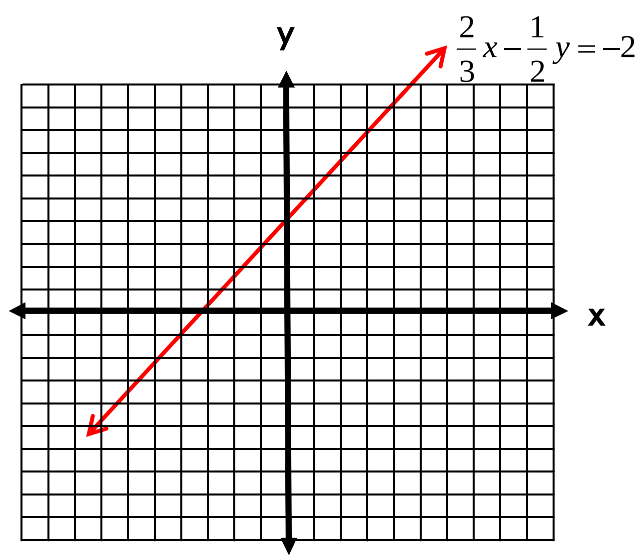
y-int: (0,Y) Let x = 0

$\frac{2}{3}(0) - \frac{1}{2}y = -2$

$-\frac{1}{2}y = -2$

$y = 4$

y-int: 4 (0,4)




**The domain and range of a linear function are all real numbers.**

**Domain:** x represents all real numbers
{-∞ < x < ∞}
(-∞ , ∞)

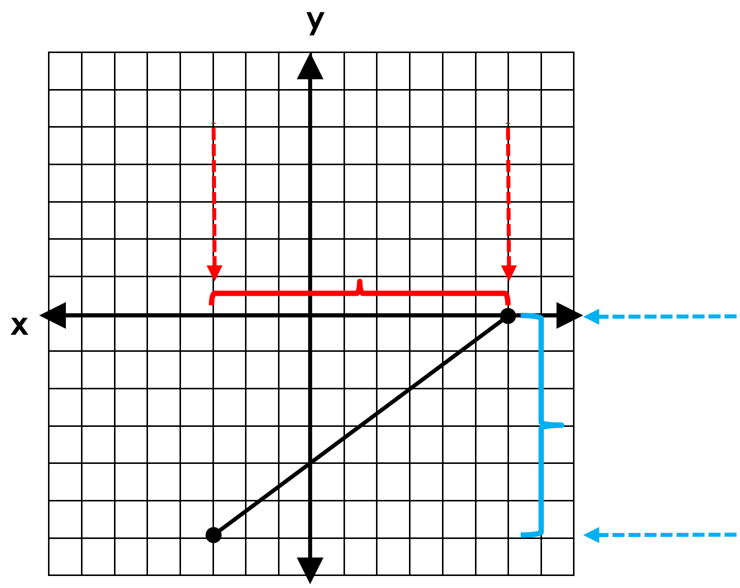
**Range:** y represents all real numbers
{-∞ < y < ∞}
(-∞ , ∞)

- A linear function may be graphed over a restricted domain.

**Ex:** Graph $y = \frac{2}{3}x - 4$ using the domain $\{x \in R \mid -3 \leq x \leq 6\}$. State the range of the function.

$y = \frac{2}{3}x - 4$

| x | y |
|---|---|
| **-3** | **-6** |
| 0 | -4 |
| 3 | -2 |
| **6** | **0** |




$y = \frac{2}{3}x - 4$

**Domain: $\{-3 \leq x \leq 6\}$**

**[-3, 6]**

***The domain consists of all real numbers between and including -3 and 6.***

**Range: $\{-6 \leq y \leq 0\}$**

**[-6, 0]**

***The range consists of all real numbers between and including -6 and 0.***

***Important Note:***

***We can define the domain and range of a function using an inequality statement or interval notation.***

C. **Graphing Horizontal and Vertical Lines**

**y = b**, *where* **b** *is any number*, is the equation of a horizontal line
**x = a**, *where* **a** *is any number*, is the equation of a vertical line (*not a function*)

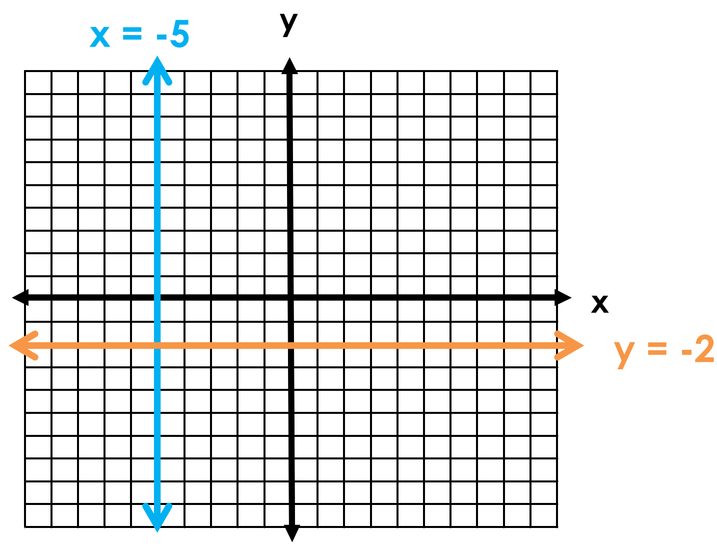
**Ex:**

y = -2 ← *horizontal line*
every output is -2

| x | y |
|---|---|
| -3 | -2 |
| 0 | -2 |
| 3 | -2 |

x = -5 ←*vertical line*
every input is -5

| x | y |
|---|---|
| -5 | -3 |
| -5 | 0 |
| -5 | 3 |




***Vertical Lines are not Functions**

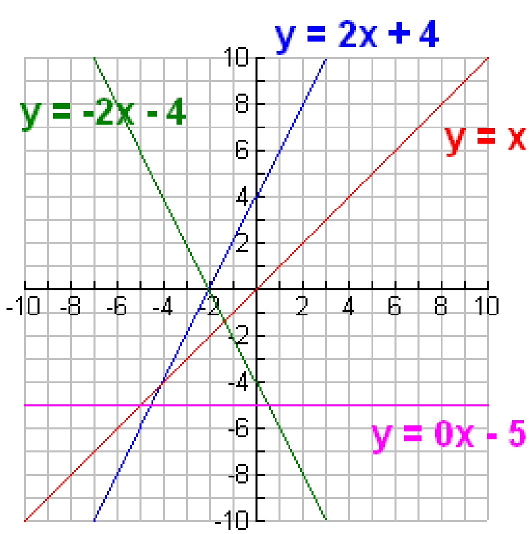
D. **Slope (Rate of Change)**

-Slope is a ratio that determines the steepness and direction of a line

-Given a graph, you can find the slope of a line using ($\frac{rise}{run}$)

-Given two points on a line, you can find the slope of a line using $\frac{\Delta y}{\Delta x}$

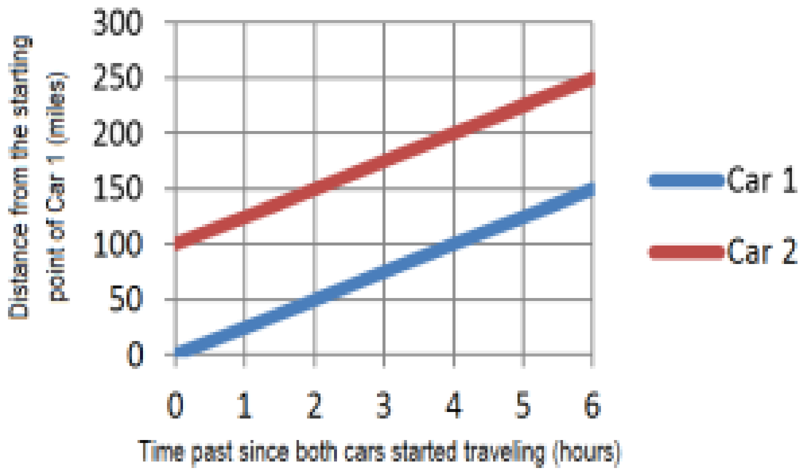
*slope (rate of change)* = $\frac{\Delta y}{\Delta x} = \frac{\text{difference in } y-\text{values}}{\text{difference in } x-\text{values}}$




***The rate of change of linear functions is constant***
***(all graphs are read from left to right)***

- If the rate of change is positive, the line slants uphill (increases).
- As the rate of change increases, the line gets steeper.
- As the rate of change gets extremely large (a very big number), the line becomes nearly vertical. If the line is vertical, the slope is undefined (because it has no horizontal change).
- As the rate of change gets smaller (closer to zero), the line loses steepness and starts to flatten. If the slope is zero, the line is horizontal.
- If the slope is negative the line slants downhill (decreases).

**Ex:** Determine the rate of change of each car.




**Car 1**: (0, 0) (2, 50)

$$\frac{\Delta y}{\Delta x} = \frac{50-0}{2-0} = \frac{50}{2} = \frac{25 miles}{1 hour}$$

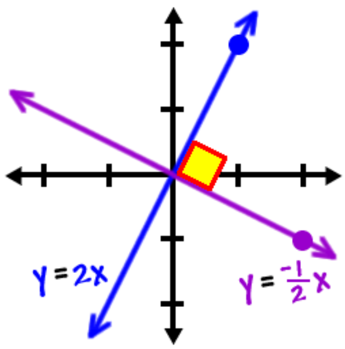
Car 1 is driving at a speed of 25mph

**Car 2**: (0,100) (2,150)

$$\frac{\Delta y}{\Delta x} = \frac{150-100}{2-0} = \frac{50}{2} = \frac{25 miles}{1 hour}$$

Car 2 is driving at a speed of 25mph

**The rates of change are equivalent because the lines are parallel.**




***Important Note:***

***Perpendicular lines have opposite reciprocal slopes as shown in the example to the left.***

***Parallel lines have the same slope but different y-intercepts.***

## E. Writing the Equation of a Line in Slope-Intercept Form

-In order to write the equation of a line in slope-intercept form, find the slope of the line and the y-intercept (0, *y*).

**Ex:** Write the equation of a line that has an x-intercept of 4 and passes through the point (2, 3).

Find the **slope** using the points (2, 3) and (4, 0)

Slope $= \frac{3-0}{2-4} = -\frac{3}{2}$ m $= -\frac{3}{2}$

Find the **y-intercept** by using the slope and 1 point on the line.

Using the slope-intercept form of an equation, substitute x, y and m with the given values (*use only one of the points*).

$y = mx + b$ Pt. (2, 3)

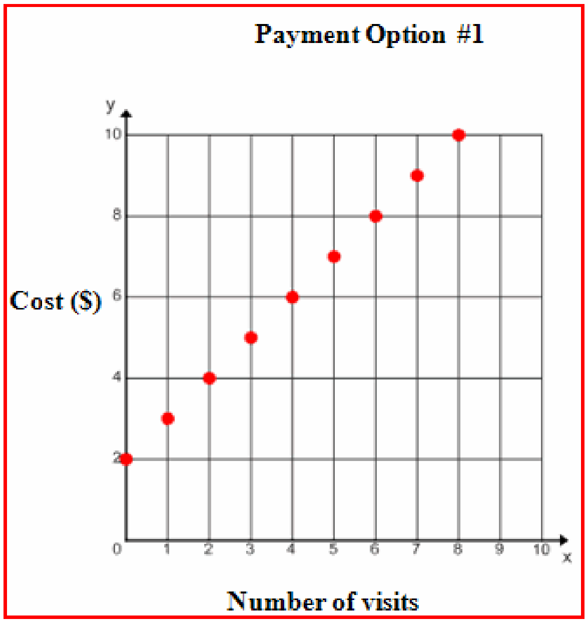
$3 = -\frac{3}{2}(2) + b$

$3 = -3 + b$

$6 = b$ *The y-intercept is 6.*

Substitute ***m*** and ***b*** into the **equation y =*mx* + *b*** **Equation: $y = -\frac{3}{2}x + 6$**

**Ex:** The local fitness center is running two different monthly payment options to attract new customers. The options are shown below. White an equation relating the number of visits (x) to the total cost (y)for each payment option.




**Payment Option #2**

Pay a fee of $5.00 and then pay $.25 every time you visit.

**Option 2:**

m = .25 ($0.25 per visit)

b = 5 ($5 initial fee)

**Equation:** y =.25x + 5

**Option 1: (2,4) (6,8)**

$$\frac{\Delta y}{\Delta x} = \frac{8-4}{6-2} = \frac{4}{4} = \frac{1}{1} = \frac{\$1}{1\ visit}$$

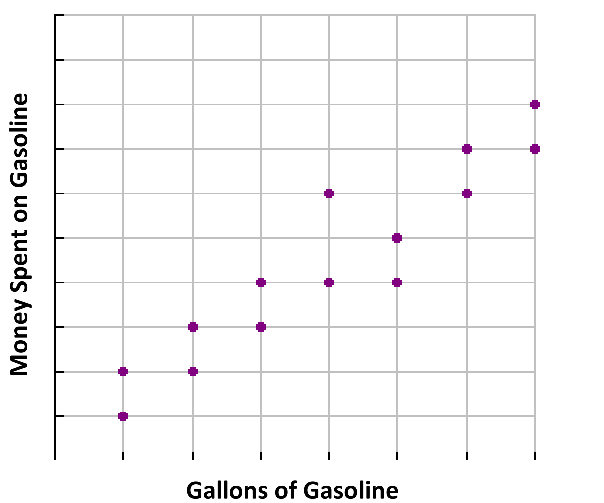
m = 1 ($1 per visit)

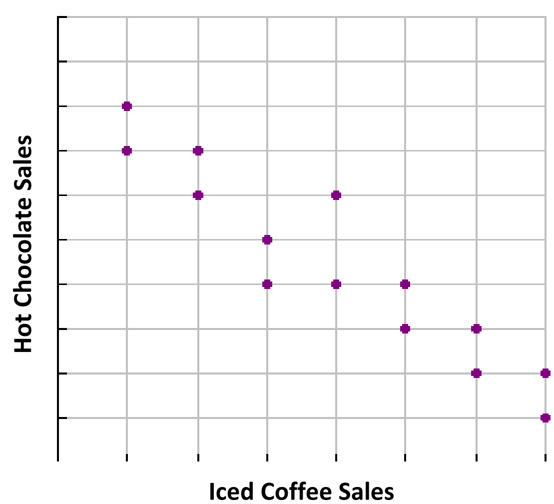
b = 2 ($2 initial fee)

**Equation:** y = 1x + 2

## C. Representing Data with Linear Models

- A scatter plot graphs bivariate data as a set of points whose coordinates correspond to the two variables.
- A correlation coefficient can help us determine the strength of a relationship between two variables.
- Not every correlation demonstrates causation.
- The line of best fit helps us predict values that do not appear in the data set.
- A residual plot tells us how well a line fits a set of data.



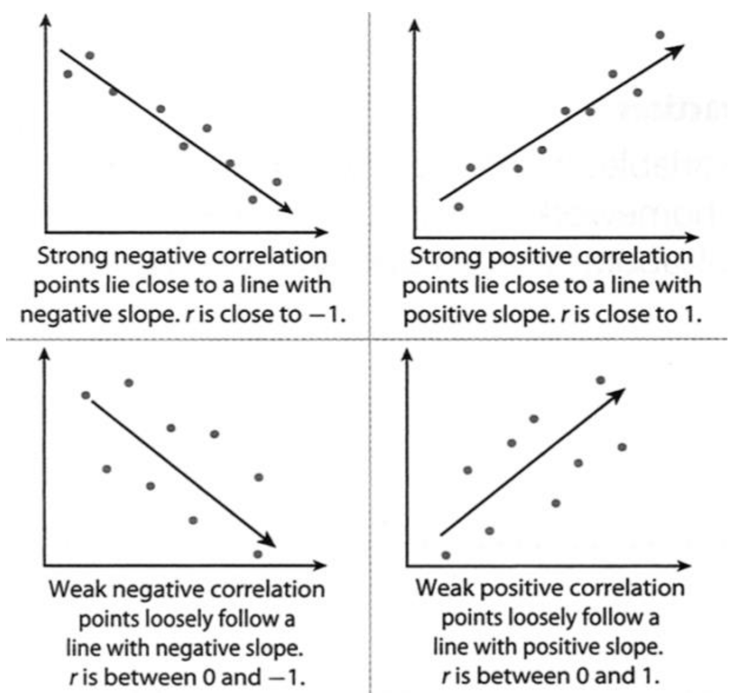




**Positive Correlation** and **Causation**. As you put more gallons of gasoline in your car, you have to spend more money. One variable directly impacts the other.

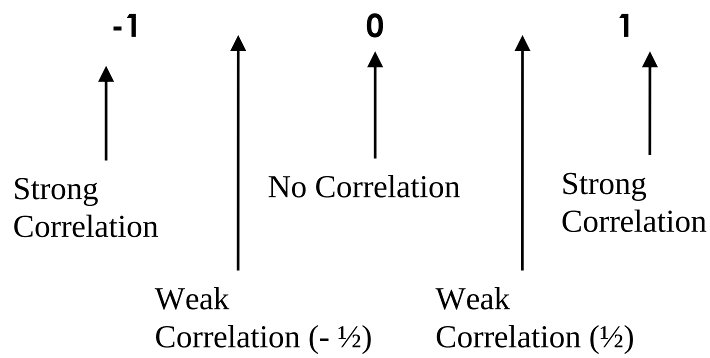
**Negative Correlation** but **No Causation**. The change of seasons is most likely the cause of iced coffee sales increasing and hot chocolate sales decreasing. One variable does not impact the other.

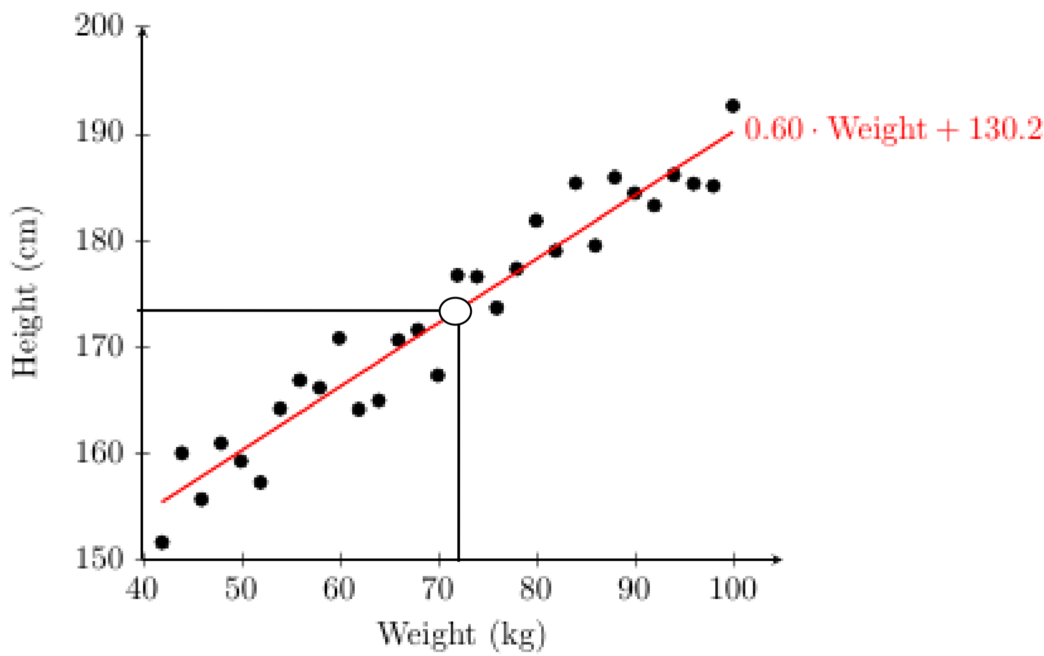

**Causation:** Bivariate data shows causation when one variable causes the other. The final grade attained in a course is impacted (caused) by the number of absences.

There is no correlation between the number of hours spent at the mall and the amount of money spent.




A correlation coefficient is represented by the variable **r**. It is a number between -1 and 1 and it informs you about the strength of the data.






-A line can be drawn to show the trend of the data set. This line is known as the **line of fit**, **regression line** or **least squares line**.

-The equation of the line helps us **interpolate** or **extrapolate** (make predictions within the set of data and beyond the set of data).

The scatter plot pictured above shows a positive correlation between a person's height and weight. The data points represent the weights and heights of different people. The linear regression equation for the line drawn above is $y = 0.6x + 130.2$ where x represents a person's weight in kg and y represents his/her corresponding height in cm.

**Ex:** Predict the height of a person who weighs 72 kg.

$y = 0.6\ (72) + 130.2$
$y = 173.4$

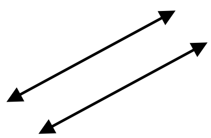
**Using our trend line, we expect a person's height to be about 173 cm if they weigh 72 kg.** ***See predicted data point on the trend line above.***

# 6) Linear Systems

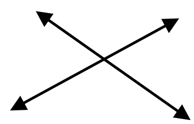
## A. Solving Linear Systems Graphically

- A system of linear equations is a collection of two or more equations in the same variable. A solution to the system is an ordered pair (x, y) that makes both equations true.
  *Example*: **System:** $2y + x = 14$ **Solution:** (2, 6)
  $y = 3x$
- There are two ways to solve a system (**graphically** or **algebraically**)
- When solving a linear system, it is possible to have one solution, no solution or infinitely many solutions.

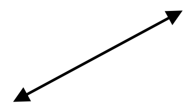
**No Solution**
$y = 2x + 5$
$y = 2x – 4$



**One Solution**
$y = -2x + 4$
$y = 3x - 2$



**Infinite Solutions**
$y = 2x + 3$
$3y = 6x + 9$



Solving a Linear System Graphically

1) Graph both lines on the coordinate plane
2) Locate the point of intersection (common solution)
3) Check the solution with both equations

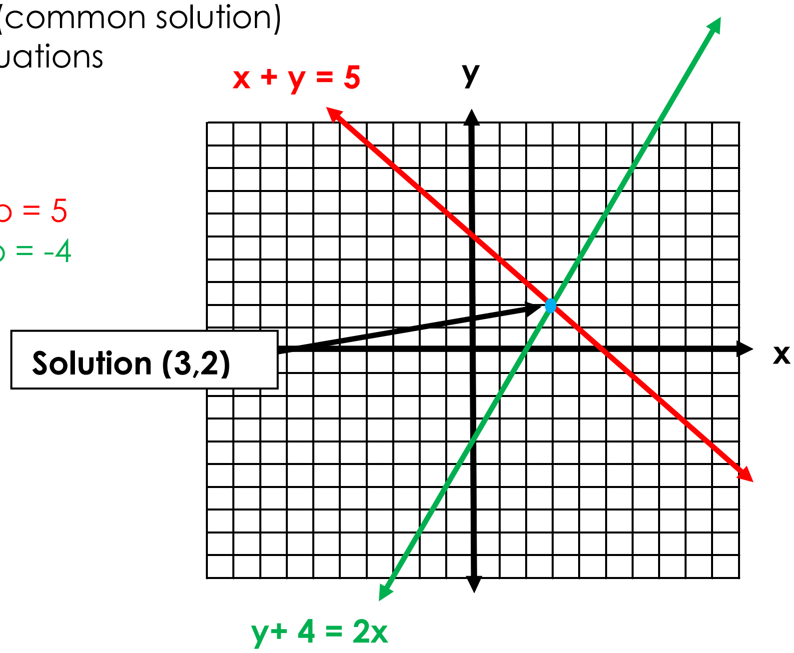
**Ex:** Solve the system graphically.

$x + y = 5$ → $y = -x + 5$  $m = -1$  $b = 5$
$y + 4 = 2x$ → $y = 2x – 4$  $m = 2$  $b = -4$

Solution (3, 2)

<u>Check</u>

| | |
|---|---|
| $x + y = 5$ | $y + 4 = 2x$ |
| $3 + 2 = 5$ | $2 + 4 = 2(3)$ |
| $5 = 5$ | $6 = 6$ |



## B. Solving a Linear System Algebraically (2 Methods)

1st Method: **Substitution**

When using the substitution method, one equation is manipulated so that "x" or "y" is isolated, then the resulting expression of that variable is substituted in the second original equation to find the value of the second variable.

**Ex:** Solve the following system using substitution. $x + y = 7$
$3x = 17 + y$

$x + y = 7 \rightarrow x = 7 - y$

$3x = 17 + y$

**Finding y**

$3x = 17 + y$
$3(7 - y) = 17 + y$
$21 - 3y = 17 + y$
$-4y = -4$
$y = 1$

**Finding x**

$x + y = 7$
$x + 1 = 7$
$x = 6$

**Solution (6, 1)**

**Check**

$x + y = 7$
$6 + 1 = 7$
$7 = 7$
$3x = 17 + y$
$3(6) = 17 + 1$
$18 = 18$

2nd Method: **Elimination**

When two equations have the same variable with opposite coefficients, we can add the equations together and eliminate the variable.

*Procedure for Solving a Linear System using Elimination*

1) Arrange both equations using the appropriate properties in order to align all terms.
2) Multiply one or both equations by a positive or negative number to obtain coefficients that are opposites.
3) Add the two equations and one variable will be eliminated.
4) Solve for the variable that remains.
5) Choose one of the original equations and substitute the value for the known variable to find the value of the other variable.
6) Check solution with both original equations.

**Ex:** Solve the following system using elimination. $5x - 2y = 10$
$2x + y = 31$

$5x - 2y = 10$
$2x + y = 31$

$5x - 2y = 10$
$2[2x + y = 31]$

$$\begin{aligned} &5x - 2y = 10 \\ + \; &\underline{4x + 2y = 62} \\ &9x + 0y = 72 \\ &9x = 72 \\ &x = 8 \end{aligned}$$

**Finding y**

$2x + y = 31$
$2(8) + y = 31$
$16 + y = 31$
$y = 15$

**Solution (8, 15)**

**Check**

$5x - 2y = 10$
$5(8) - 2(15) = 10$
$40 - 30 = 10$
$10 = 10$
$4x + 2y = 62$
$4(8) + 2(15) = 62$
$32 + 30 = 62$
$62 = 62$

## C. <u>Using Systems to Solve Word Problems</u>

*Procedure for setting up a 2 variable System of Equations*

1) Identify each unknown quantity and represent each one with a different variable.
2) Translate the verbal sentences into *two* equations.
3) Solve the system algebraically.
4) Check the answers using the words of the problem. Ask yourself, does my answer make sense?

**Ex:** Together Evan and Denise have 28 books. If Denise has four more than Evan, how many books does each person have? Use a system to solve the problem.

Let $x$ = the number of books Evan has

Let $y$ = the number of books Denise has

$x + y = 28$
$y = x + 4$

Using the substitution method $\rightarrow$ $x + (x + 4) = 28$

$2x + 4 = 28$
$2x = 24$
$x = 12$

$x + y = 28$
$12 + y = 28$
$y = 16$

**Evan has 12 books and Denise has 16 books**