Essential Question: How do we solve equations with rational expressions?

Do Now: Solve the following equations.

(a) $33 = \dfrac{8}{s} + 9$

$ -9 \phantom{\dfrac{8}{s}} --9$

$\dfrac{24}{1} = \dfrac{8}{s}$

$\dfrac{24s}{24} = \dfrac{8}{24} \qquad s = \dfrac{1}{3}$

(b) $\dfrac{x}{(x+4)} = \dfrac{2}{1}$

$x = 2(x+4)$

$x = 2x + 8$

$-2x \quad -2x$

$\dfrac{-x}{-1} = \dfrac{8}{-1} \qquad x = -8$

RATIONAL EXPRESSIONS   &   RATIONAL EQUATIONS

an expression that is the ratio of two polynomials
$\dfrac{3x}{2} \qquad \dfrac{x+5}{x-2}$
$\dfrac{x-2}{4}$

an equation with one or more rational expressions
$3x + \dfrac{x}{3} = 5 \qquad \dfrac{x}{x+3} = \dfrac{8}{x+6}$
$\dfrac{7}{x} + \dfrac{3}{4} = \dfrac{5}{x}$

When solving rational equations, identify all values of the variable that are **non-permissible values** (sometimes referred to as **extraneous solutions**). Non-permissible values are <u>values that make the denominator zero</u>.

What are the non-permissible values of the equations in today's Do Now?

$33 = \dfrac{8}{s} + 9 \qquad s \neq \underline{\ \ 0\ \ }$

$\dfrac{x}{x+4} = 2 \qquad x \neq \underline{\ -4\ }$

Determine whether the given value of x is a non-permissible value for the rational equation. Explain how you know.
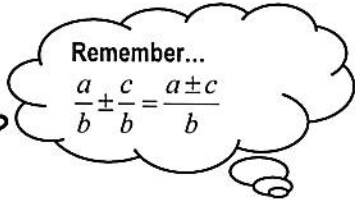
a) $\dfrac{3x}{x+6} = 2; \ x = 6$

x is a permissible value when x is replaced by 6, the denominator is 12

b) $\dfrac{x+4}{x-4} = -3; \ x = 4$

4 is a non-permissible value when x is replaced by 4, the denominator equals 0 (and the fraction would be undefined)

SOLVING RATIONAL EQUATIONS

How do we solve an equation containing rational expressions?

> Remember...
> $$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$$

(a) Consider the following equation...

$$\frac{5x}{4} + \frac{x}{4} = 12$$

- Is the equation a proportion? **NO**
- How would you solve this equation?
  - combine numerators
  - set up proportion
  - cross multiply and solve

$$\frac{5x + x}{4} = 12$$

$$\frac{6x}{4} = \frac{12}{1} \qquad 6x = 4(12)$$
$$6x = 48$$
$$x = 8$$

(b) Let's try another example.

$$\frac{2x}{5} + 1 = \frac{13}{5}$$

- Is the equation a proportion? **No**
- How would you solve this equation?
  - make the 1 a fraction with a denominator of 5
  - combine fractions
  - solve the proportion

$$\frac{2x}{5} + \frac{5}{5} = \frac{13}{5}$$

$$\frac{2x + 5}{5} = \frac{13}{5} \qquad (2x+5)5 = 13(5)$$
$$2x + 5 = 13$$
$$2x = 8$$
$$x = 4$$

(c) What about this equation?

$$\frac{3x}{2} - \frac{2x}{3} = 5$$

- Is the equation a proportion? **No**
- How would you solve this equation?

In order to add and subtract fractions, the denominators must BE THE SAME!

If denominators are not the same,	$$\frac{3x}{2} - \frac{2x}{3} = 5$$
• you must find the **LCD**	LCD : 6
• write an **equivalent fraction** with the LCD (multiply by a *"form of one"* – **FOO**)	$$\frac{3}{3}\left(\frac{3x}{2}\right) - \frac{2}{2}\left(\frac{2x}{3}\right) = 5$$
• add/subtract rational expressions to create a proportion	$$\frac{9x}{6} - \frac{4x}{6} = 5$$
• cross multiply	$$\frac{5x}{6} = \frac{5}{1}$$
• solve the equation	
• CHECK your answer!	$$30 = 5x$$ $$6 = x$$

Let's try solving a few more rational equations. Check your solution! using <u>FOO</u> (form of one)

1) $\dfrac{3x}{4} - \dfrac{x}{4} = -5$ LCD: 4

$\dfrac{2x}{4} = \dfrac{-5}{1}$

$2x = -5(4)$

$2x = -20$

$\boxed{x = -10}$

2) $\dfrac{x}{6} - \dfrac{2}{3} = \dfrac{5}{6}$ LCD: 6

$\dfrac{x}{6} - \dfrac{2}{2}\left(\dfrac{2}{3}\right) = \dfrac{5}{6}$

$\dfrac{x}{6} - \dfrac{4}{6} = \dfrac{5}{6}$

$\dfrac{x-4}{6} = \dfrac{5}{6}$

$6(x-4) = 6(5)$

$6x - 24 = 30$

$6x = 54$

$\boxed{x = 9}$

3) $\dfrac{x+3}{5} - \dfrac{3x}{10} = 7$ LCD: 10

$\dfrac{2}{2}\left(\dfrac{x+3}{5}\right) - \dfrac{3x}{10} = 7$

$\dfrac{2(x+3)}{10} - \dfrac{3x}{10} = 7$

$\dfrac{2x+6-3x}{10} = 7$

$\dfrac{-x+6}{10} = \dfrac{7}{1}$

$7(10) = -x+6$

$70 = -x+6$

$\quad\; -6 \qquad -6$

$\dfrac{64}{-1} = \dfrac{-x}{-1}$ $\boxed{x = -64}$

4) $\dfrac{3x}{7} + 1 = \dfrac{2}{5}$ LCD: 35

$\dfrac{5}{5}\left(\dfrac{3x}{7}\right) + \dfrac{35}{35}(1) = \dfrac{2}{5}\left(\dfrac{7}{7}\right)$

$\dfrac{15x}{35} + \dfrac{35}{35} = \dfrac{14}{35}$

↗ if all the denominators are the same, write the numerators as an equation

$15x + 35 = 14$

$\quad\; -35 \qquad -35$

$\dfrac{15x}{15} = \dfrac{-21}{15}$

$\boxed{x = \dfrac{-7}{5}}$

5) $\dfrac{3x}{4} - \dfrac{x-1}{2} = \dfrac{x}{2}$ LCD : 4

6) $\dfrac{x}{5} - \dfrac{2x+1}{3} = -5$ LCD : 15

$\dfrac{3x}{4} - \dfrac{2}{2}\left(\dfrac{x-1}{2}\right) = \left(\dfrac{x}{2}\right)\dfrac{2}{2}$

$\dfrac{3}{3}\left(\dfrac{x}{5}\right) - \dfrac{5}{5}\left(\dfrac{2x+1}{3}\right) = \dfrac{-5}{1}\left(\dfrac{15}{15}\right)$

$\dfrac{3x}{4} - \dfrac{2(x-1)}{4} = \dfrac{2x}{4}$

$\dfrac{3x}{15} - \dfrac{5(2x+1)}{15} = \dfrac{-75}{15}$

$3x - 2(x-1) = 2x$

$3x - 2x + 2 = 2x$

$x + 2 = 2x$

$\boxed{2 = x}$

$3x - 5(2x+1) = -75$

$3x - 10x - 5 = -75$

$-7x - 5 = -75$

$+5 \qquad +5$

$\dfrac{-7x}{-7} = \dfrac{-70}{-7}$

$\boxed{x = 10}$

or

$\dfrac{3}{3}\left(\dfrac{x}{5}\right) - \dfrac{5}{5}\left(\dfrac{2x+1}{3}\right) = -5$

$\dfrac{3x}{15} - \dfrac{5(2x+1)}{15} = -5$

$\dfrac{3x - 10x - 5}{15} = -5$

$-75 = -7x - 5$

$+5 \qquad\quad +5$

$\dfrac{-70}{-7} = \dfrac{-7x}{-7}$ $\boxed{x = 10}$

$\dfrac{-7x - 5}{15} = \dfrac{-5}{1}$

The TAKE AWAY

To solve rational equations, combine fractions and create a proportion. Combining fractions requires a common denominator. In order to create fractions with a common denominator, multiply each fraction by a FOO (_Form of One_). When a fraction is multiplied by a FOO, an equivalent fraction is created.