

Essential Questions: How do we recognize equations that have no solution? How do we recognize equations that have infinitely many solutions? How do we recognize equations that have the same solution set?

Do Now: Solve each equation.

A) $3(x+2) = 3x+6$

$$\begin{array}{r} 3x+6 = 3x+6 \\ -3x \quad -3x \\ \hline 6 = 6 \end{array}$$

B) $3x+2-2x = \frac{1}{2}(2x+8)$

$$\begin{array}{r} x+2 = x+4 \\ -x \quad -x \\ \hline 2 \neq 4 \end{array}$$



Not every equation has one solution.

There are equations that exist that have **infinitely many solutions**.

There are equations that exist that have **no solution**.

Example:

$$5(2x-4) = 3(3x-6) + x - 2$$

$$10x - 20 = 9x - 18 + x - 2$$

$$10x - 20 = 10x - 20 \leftarrow \text{Both sides are the same}$$

$$10x = 10x$$

$0 = 0$ This equation has **infinitely many solutions**.
 $x = \text{all real numbers}$

Example:

$$2(x+4) + 3 = 2x + 6$$

$$2x + 8 + 3 = 2x + 6$$

$$2x + 11 = 2x + 6 \leftarrow \text{This doesn't make sense}$$

$$11 \neq 6$$

This equation has **no solution**.

Decide if each equation below has one, none or infinitely many solutions.

1. $5x - 1 - 4x = 3 + x - 4$

$$x - 1 = x - 1$$

infinitely many solutions

[the expressions on each side are identical]

2. $\frac{1}{4}(8x - 16) = 5x - 11$

$$\begin{array}{r} 2x - 4 = 5x - 11 \\ -2x \quad -2x \\ \hline \end{array}$$

$$\begin{array}{r} -4 = 3x - 11 \\ +11 \quad +11 \\ \hline \end{array}$$

$$\frac{7}{3} = \frac{3x}{3}$$

$$x = \frac{7}{3}$$

one solution

3. $9(x - 1) = 3x + 5 + 6x$

$$\begin{array}{r} 9x - 9 = 9x + 5 \\ -9x \quad -9x \\ \hline \end{array}$$

$$-9 \neq 5$$

no solutions

-9 will never equal 5

4. Consider the equation: $4y + 5 - y = 3y - 8 + 12$

$$4y + 5 - y = 3y - 8 + 12$$

A) Determine if the equation has one, none or infinite solutions.

$$3y + 5 = 3y + 4$$

$$5 \neq 4$$

B) How can the equation be changed so that it has an infinite number of solutions? *no solution*

add one to the right side

$$4y + 5 - y = 3y - 8 + 12 + 1$$

$$3y + 5 = 3y + 5$$

identical expressions

Equivalent Equations

Equations that have the same solution set are equivalent.

$$\left[\begin{array}{l} 2x + 5 = 11 \text{ and } 10x + 25 = 55 \text{ are equivalent equations} \\ \begin{array}{r} -5 \quad -5 \\ \hline 2x = 6 \\ \hline x = 3 \end{array} \qquad \begin{array}{r} -25 \quad -25 \\ \hline 10x = 30 \\ \hline x = 3 \end{array} \leftarrow \text{same solution set} \end{array} \right.$$

Do you notice anything about the equations $2x + 5 = 11$ and $10x + 25 = 55$?

when you multiply the terms first equation by 5, it becomes the second equation

How can we determine if two or more equations are equivalent and share the same solution set?

A) $2x + 3 = 13 - 5x$
 $+5x \qquad +5x$

$$7x + 3 = 13$$

$$-3 \quad -3$$

$$7x = 10$$

$$x = \frac{10}{7}$$

B) $6 + 4x = -10x + 26$
 $+10x \quad +10x$

$$6 + 14x = 26$$

$$-6 \quad -6$$

$$\frac{14x}{14} = \frac{20}{14}$$

$$x = \frac{10}{7}$$

or recognize if you divide by it looks like the equation in A

Using the properties of real numbers, determine which of the following equations have the same solution set. Solve the equations to check your response.

A. $15(2x + 3) + 97 = 110 - 5x$
 $\qquad -97 \quad -97$

$$15(2x + 3) = 13 - 5x$$

B. $x - 5 = 3x + 7$

C. $\frac{9x + 21}{3} = \frac{3x - 15}{3}$

$$3x + 7 = x - 5$$

D. $15(2x + 3) = -5x + 13$

identical (with arrow from A to C)
identical (with arrow from B to D)



TODAY'S TAKE AWAY...

Some equations have one, none or infinitely many solutions.

Equivalent equations have the same solution set.