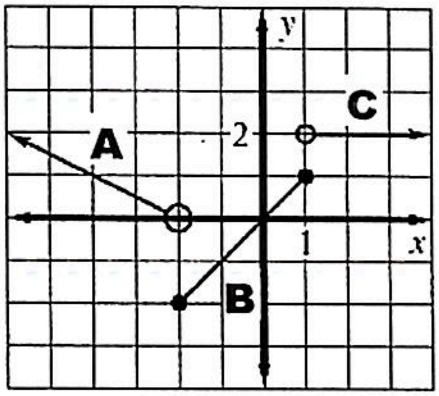
**Essential Questions:** What are piecewise functions and how are they graphed? How do we define a piecewise function?

**Do Now:**
State the domain for each line graphed below.




inequality notation | interval notation

A. $x < -2$ $(-\infty, -2)$

B. $-2 \leq x \leq 1$ $[-2, 1]$

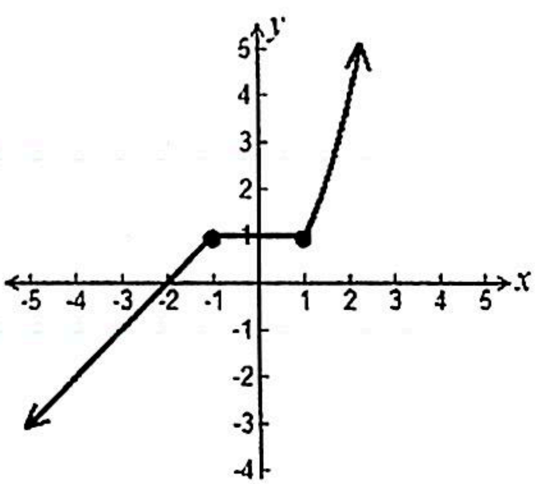
C. $x > 1$ $(1, \infty)$

# PIECEWISE FUNCTIONS

We have seen many graphs that are expressed as single equations and are continuous over a domain of real numbers. There are also graphs that are defined by "different equations". These graphs may be continuous, or they may contain "breaks". Because these graphs tend to look like "pieces" glued together to form a graph, they are referred to as **piecewise functions.**

A piecewise defined function is a function defined by at least two equations ("pieces"), each of which applies to a different part of the domain. Piecewise defined functions can take on a variety of forms. Their "pieces" may be all linear, or a combination of functional forms (such as constant, linear, quadratic, cubic, square root, cube root, exponential, etc.).

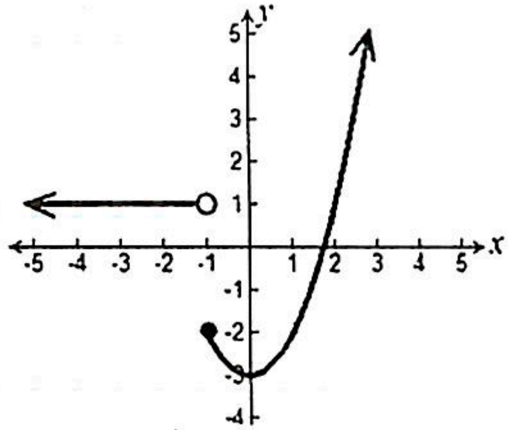
**Example:**




$$f(x)=\begin{cases} x+2; & x \leq -1 \\ 1; & -1 < x < 1 \\ x^2; & x \geq 1 \end{cases}$$

The piecewise function shown in this example is ***continuous*** (there are no "gaps" or "breaks" in the graph).

In this example, the domain is all real numbers since all $x$-values have a value.

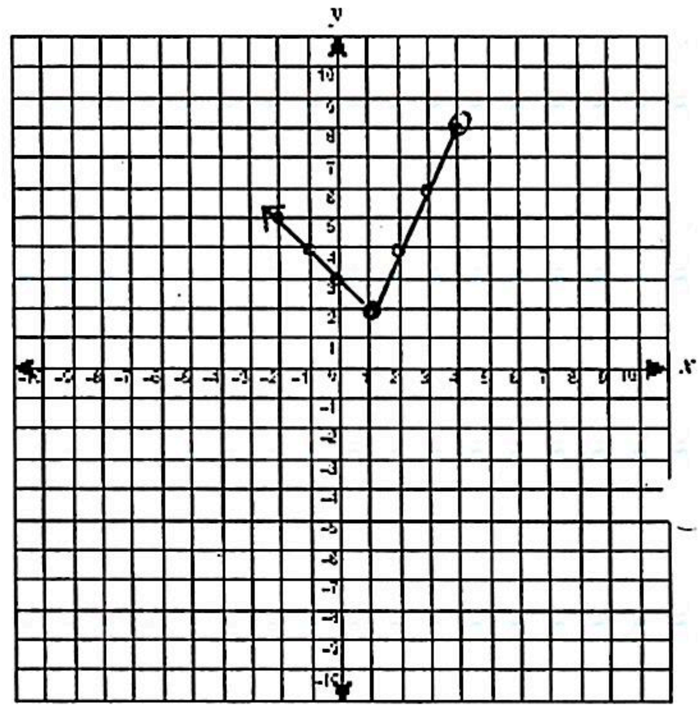


1. (a) domain: $\{x \mid x \in \mathbb{R}\}$   $(-\infty, \infty)$

(b) range: $\{y \mid y \geq -3\}$   $[-3, \infty)$

**Graph the given piecewise functions below.**

2. $f(x) = \begin{cases} -x+3 & x < 1 \\ 2x & 1 \leq x < 4 \end{cases}$



$f(x) = -x+3$

| | x | f(x) |
|---|---|---|
| ↑ | -2 | 5 |
| | -1 | 4 |
| | 0 | 3 |
| o | 1 | 2 |

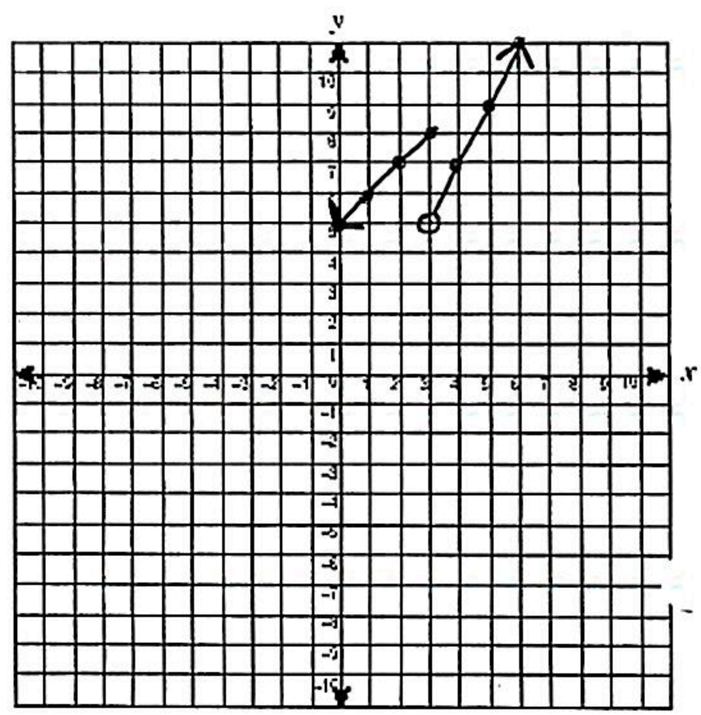
$f(x) = 2x$

| x | f(x) | |
|---|---|---|
| 1 | 2 | • |
| 2 | 4 | |
| 3 | 6 | |
| 4 | 8 | o |

domain: $\{x \mid x < 4\}$

range: $\{y \mid y \geq 2\}$

3. $g(x) = \begin{cases} x + 5, \text{ if } x \leq 3 \\ 2x - 1, \text{ if } x > 3 \end{cases}$



$g(x) = x+5$

| | x | g(x) |
|---|---|---|
| • | 3 | 8 |
| | 2 | 7 |
| | 1 | 6 |
| ↓ | 0 | 5 |

$g(x) = 2x-1$

| | x | g(x) |
|---|---|---|
| o | 3 | 5 |
| | 4 | 7 |
| | 5 | 9 |
| ↓ | 6 | 11 |

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

Solve the system.

$$f(x) = \begin{cases} 2x+1 & x \geq 1 \\ x^2+3 & x < 1 \end{cases}$$

$g(x) = x + 5$

$m = 1$

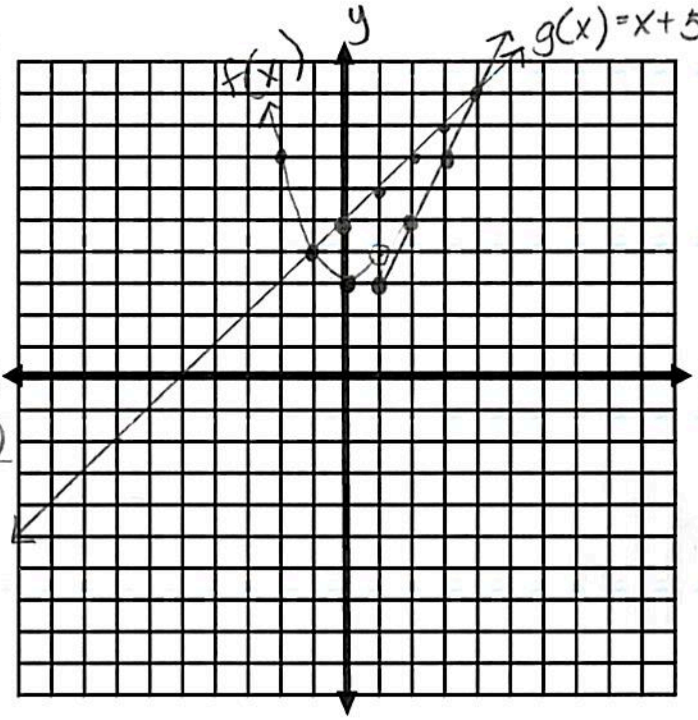
$b = 5$

$f(x) = 2x+1$

| | x | f(x) |
|---|---|---|
| closed | 1 | 3 |
| | 2 | 5 |
| | 3 | 7 |
| arrow | 4 | 9 |

$f(x) = x^2+3$

| | x | f(x) |
|---|---|---|
| arrow | -2 | 7 |
| | -1 | 4 |
| | 0 | 3 |
| open | 1 | 4 |




Solutions:

$(-1, 4)$ and $(4, 9)$

## Defining Piecewise Functions

- Write an expression for each piece graphed over its domain.
- Write a definition for the graph, which is done by identifying the different domains shown in the graph.
- Remember: ○ for < and >

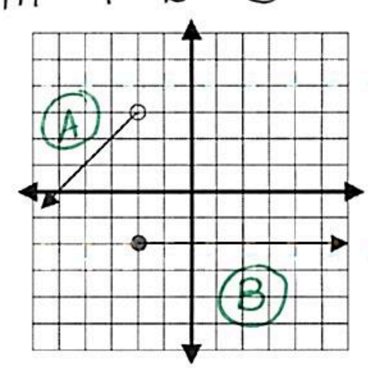
  ● for ≤ and ≥

$m = 1 \quad b = 5$

Write an equation for the piecewise function whose graph is shown.

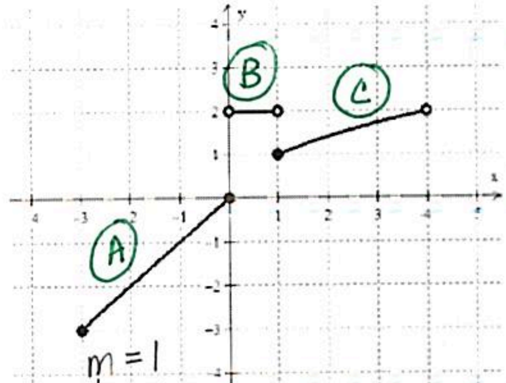
a)

$$f(x) = \begin{cases} x+5 & x < -2 \quad \text{A} \\ -2 & x \geq -2 \quad \text{B} \end{cases}$$



b)

$$g(x) = \begin{cases} x & -3 \le x \le 0 \quad \text{(A)} \\ 2 & 0 < x < 1 \quad \text{(B)} \\ \sqrt{x} & 1 \le x < 4 \quad \text{(C)} \end{cases}$$




$m = 1$
$b = 0$

TAKE AWAY

Piecewise functions are made up of different pieces of functions. Each piece is represented by a different equation with a restricted domain.

**IT'S YOUR TURN NOW**

Graph each piecewise function.

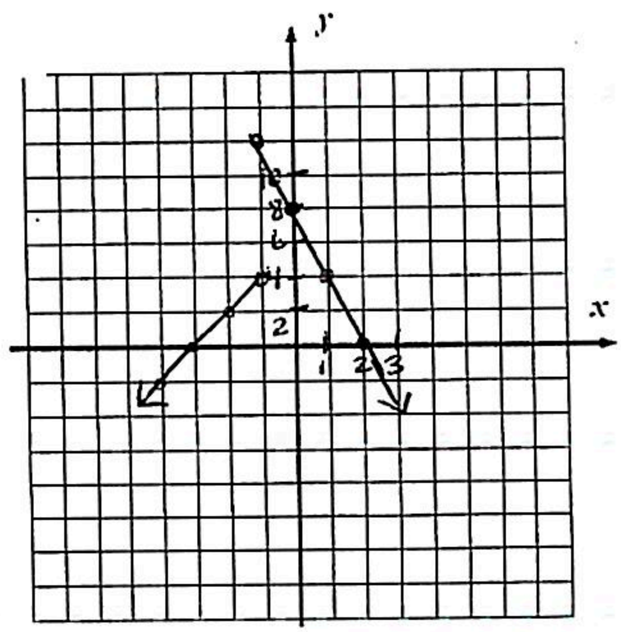
(a) $f(x) = \begin{cases} 2x+6 & x < -1 \\ -4x+8 & x \ge -1 \end{cases}$

$f(x) = 2x + 6$

| | x | f(x) |
|---|---|---|
| o | -1 | 4 |
| | -2 | 2 |
| | -3 | 0 |
| ↓ | -4 | -2 |

$f(x) = -4x + 8$

| | x | f(x) |
|---|---|---|
| • | -1 | 12 |
| | 0 | 8 |
| | 1 | 4 |
| ↓ | 2 | 0 |




domain: $(-\infty, \infty)$

range: $(-\infty, 12]$