

Essential Question: How can we determine if a function is linear, exponential or quadratic?

Do Now:

- a) When tables are used to model functions, we typically have just a few sample values of the function and therefore have to do some detective work to figure out what the function might be. What type of function (*linear, exponential or quadratic*) do you think each table models? *Be ready to justify your response.*

$x$	$f(x)$
0	6
1	12
2	18
3	24
4	30
5	36

linear  
has a common  
difference of 6

$x$	$g(x)$
0	0
1	14
2	24
3	30
4	32
5	30

quadratic  
increases and  
then starts to  
decrease

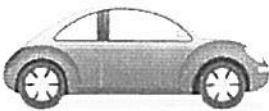
$x$	$h(x)$
0	1
1	3
2	9
3	27
4	81
5	243

exponential  
has a common  
ratio of 3

- b) Three cars start traveling at the same time. The distance traveled in  $t$  minutes is  $y$  miles. Graph the distances of each car over the first minute on your calculator. Use the indicated window setting.


**Window Setting**  
Xmin = 0  
Xmax = 1  
Xscl = 0.025  
Ymin = 0  
Ymax = 1  
Yscl = 0.025

Car 1



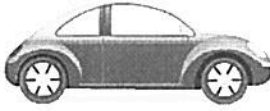
$t$	$y_1 = t$
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Car 2



$t$	$y_2 = 2^t - 1$
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Car 3



$t$	$y_3 = t^2$
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compare the table of values

Which car is moving at a constant rate? Explain your reasoning.

car 1 → constant rate of 1  
(graph is linear)

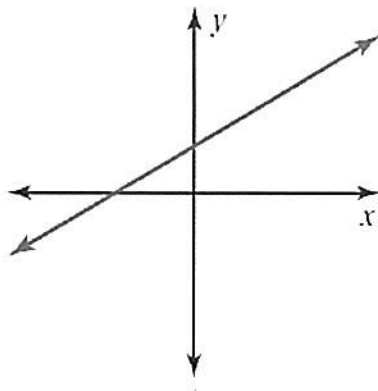
Which car accelerated the most during the first minute? Explain your reasoning.

car 3 →

minutes	Car 1	Car 2	Car 3
0.2	0.2	0.1487	0.04
1	1	1	1

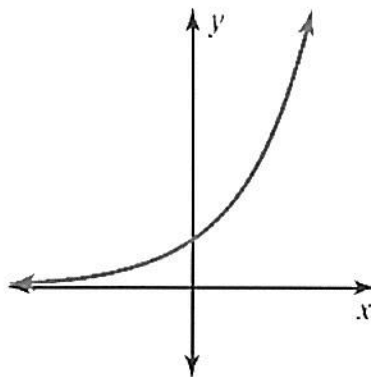
at the beginning, this car was behind the other cars. By the end of a minute, it caught up. To catch up, it accelerated the most.

## Linear Function



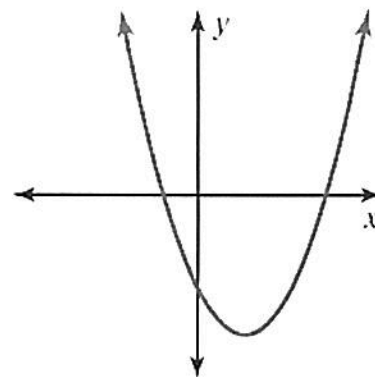
Line  
 $y = mx + b$

## Exponential Function



Curve  
 $y = ab^x$

## Quadratic Function



Parabola  
 $y = ax^2 + bx + c$

### Identifying Functions Using Differences or Ratios

One method for identifying functions is to look at the *difference* or the *ratio* of different values of the dependent variable.

If the *difference* between values of the dependent variable is the same each time we change the independent variable by the same amount, then the function is linear.

		+1	+1	-1	+1	
		↘	↘	↘	↘	
x	-2	-1	0	1	2	
y	1	3	5	7	9	
		↖	↖	↖	↖	
		-2	+2	-2	+2	

The y-values have a common *difference* of 2.

If the *ratio* between values of the dependent variable is the same each time we change the independent variable by the same amount, then the function is exponential.

		+1	+1	-1	+1	
		↘	↘	↘	↘	
x	-2	-1	0	1	2	
y	1	2	4	8	16	
		↖	↖	↖	↖	
		×2	×2	×2	×2	

The y-values have a common *ratio* of 2.

Differences can also be used to identify quadratic functions. For a quadratic function, when we increase the  $x$  values by the same amount, the difference between  $y$  values will *not* be the same. However, the difference of the differences of the  $y$  values will be the same.

		+1	+1	+1	+1	
$x$	-2	-1	0	1	2	
$y$	-1	-2	-1	2	7	
		-1	+1	+3	+5	← First differences
		+2	+2	+2		← Second differences

Tell whether the table of values represents a linear, an exponential, or a quadratic function.

1.

$x$	-2	-1	0	1	2
$y$	0	0.5	1	1.5	2

$\curvearrowright$   
 $+0.5$   $\curvearrowright$   
 $+0.5$   
 linear

2.

$x$	-1	0	1	2	3
$y$	0.2	1	5	25	125

exponential  
 (x by 5)

3.

$x$	-2	-1	0	1	2
$y$	0.75	1.5	3	6	12

exponential  
 (multiply by 2)

4.

$x$	2	3	4	5	6
$y$	2	4.5	8	12.5	18

$\curvearrowright$   
 $2.5$   $\curvearrowright$   
 $3.5$   $\curvearrowright$   
 $4.5$   $\curvearrowright$   
 $5.5$   
 $\curvearrowright$   
 $+1$   $\curvearrowright$   
 $+1$   $\curvearrowright$   
 $+1$   
 quadratic

5. Match the function to the situation.

A.  $p(x) = -16x^2 + 30x + 160$

B.  $f(x) = 10x$

C.  $q(x) = 2^x$

C The population of bacteria doubled every month, and the total population vs. time was recorded.

A A ball was launched upward from the top of a building, and the vertical distance of the ball from the ground vs. time was recorded. *force of gravity brings it down*

B Melvin saves the same amount of money every month. The total amount saved after each month was recorded.

6. Analyze these data sets. Match the function on the right to the table. Use the function to fill in the missing data.

Table A

x	y
0	6
1	10
2	14
3	18
4	22
5	26

linear

$$r(x) = 4x + 6$$

Table B

x	y
0	6
1	15
2	18
3	15
4	6
5	-9

quadratic

Table C

x	y
-1	$\frac{1}{6}$
0	1
1	6
2	36
3	216
4	1296

exponential

Table D

x	y
-1	0
0	6
1	8
2	6
3	0
4	-10
5	-24

quadratic

Equations:

$$f(x) = 6^x \quad (C)$$

$$h(x) = -3(x - 2)^2 + 18 \quad (B)$$

$$g(x) = -2(x + 1)(x - 3) \quad (D)$$

$$r(x) = 4x + 6 \quad (A)$$

# TAKE AWAY!

How can I tell the difference between linear, exponential and quadratic functions from a table of values?

A common difference can be calculated if the function is linear.

A common ratio can be calculated if the function is exponential.

A common second difference can be calculated if the function is quadratic.