

**Essential Question:** How do we solve quadratic equations by completing the square?

**Do Now:**

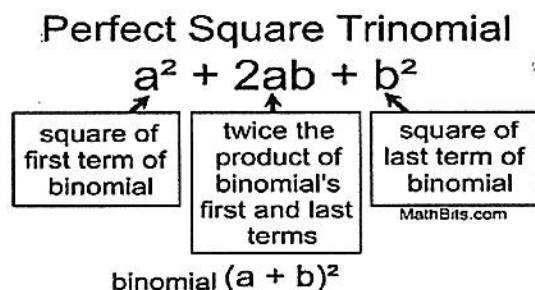
a) Simplify  $(x - 5)(x - 5)$

$$x^2 - 10x + 25$$

b) Factor  $x^2 + 8x + 16$

$$(x + 4)(x + 4)$$

The polynomial expressions in the Do Now are Perfect Square Trinomials:



Find the value of  $c$  that makes each quadratic trinomial a perfect square.

a)  $x^2 + 6x$  +9  $(6 \div 2)^2$       b)  $x^2 - 2x$  +1  $(-2 \div 1)^2$       c)  $x^2 - 14x$  +49  $(-14 \div 2)^2$

We can solve Quadratic Equations using a method called Completing The Square. It requires creating perfect square trinomials.

|  |   |
|--|---|
| 1. Be sure that the coefficient of the highest power is one. If it is not, divide each term by that value to create a leading coefficient of one.  | $x^2 + 8x - 4 = 0$  |
| 2. Move the constant term to the right hand side.  | $x^2 + 8x = 4$  |
| 3. Prepare to add the needed value to create the perfect square trinomial. Be sure to balance the equation. The boxes may help you remember to balance.  | $x^2 + 8x + \square = 4 + \square$  |
| 4. To find the needed value for the perfect square trinomial, take half of the coefficient of the <i>middle term</i> (x-term), square it, and add that value to both sides of the equation.<br><br>Take half and square<br>↓<br>$x^2 + 8x + \square = 4 + \square$ | $x^2 + 8x + \boxed{16} = 4 + \boxed{16}$  |
| 5. Factor the perfect square trinomial.  | $(x + 4)^2 = 20$  |
| 6. Take the square root of each side and solve. Remember to consider both plus and minus results.  | $x + 4 = \pm\sqrt{20}$<br>$x = -4 \pm \sqrt{20} = -4 \pm 2\sqrt{5}$<br>$x = -4 + 2\sqrt{5}$<br>$x = -4 - 2\sqrt{5}$ |

Solve each quadratic equation below by completing the square.

(1)  $x^2 - 6x - 14 = 0$

$$\begin{aligned}
 & x^2 - 6x = 14 \\
 & (-6 \div 2)^2 \quad \swarrow \quad \searrow \\
 & x^2 - 6x + 9 = 14 + 9 \\
 & (x-3)^2 = 23 \\
 & (x-3)^2 \\
 & \sqrt{(x-3)^2} = \pm \sqrt{23} \\
 & x-3 = \pm \sqrt{23} \\
 & x = 3 \pm \sqrt{23} \\
 & \left\{ 3 + \sqrt{23}, 3 - \sqrt{23} \right\}
 \end{aligned}$$

(3)  $\frac{2x^2}{2} + \frac{16x}{2} = \frac{4}{2}$

$$\begin{aligned}
 & x^2 + 8x = 2 \\
 & (8 \div 2)^2 \quad \swarrow \quad \searrow \\
 & x^2 + 8x + 16 = 2 + 16 \\
 & (x+4)^2 = 18 \\
 & \sqrt{(x+4)^2} = \pm \sqrt{18} \\
 & x+4 = \pm \sqrt{9} \sqrt{2} \\
 & x+4 = \pm 3\sqrt{2} \\
 & x = -4 \pm 3\sqrt{2} \\
 & \left\{ -4 + 3\sqrt{2}, -4 - 3\sqrt{2} \right\}
 \end{aligned}$$

(2)  $x^2 + 2x - 7 = 0$

$$\begin{aligned}
 & x^2 + 2x = 7 \\
 & (2 \div 2)^2 \quad \swarrow \quad \searrow \\
 & x^2 + 2x + 1 = 7 + 1 \\
 & (x+1)^2 = 8 \\
 & \sqrt{(x+1)^2} = \pm \sqrt{8} \\
 & x+1 = \pm \sqrt{8} \\
 & x = -1 \pm \sqrt{8} \\
 & x = -1 \pm \sqrt{4} \sqrt{2} \\
 & x = -1 \pm 2\sqrt{2} \\
 & \left\{ -1 + 2\sqrt{2}, -1 - 2\sqrt{2} \right\}
 \end{aligned}$$

(4)  $\frac{-x^2}{-1} + \frac{8x}{-1} = \frac{-9}{-1}$

$$\begin{aligned}
 & x^2 - 8x = 9 \\
 & x^2 - 8x + 16 = 9 + 16 \\
 & (x-4)^2 = 25 \\
 & \sqrt{(x-4)^2} = \pm \sqrt{25} \\
 & x-4 = \pm 5 \\
 & x = 4 \pm 5 \\
 & \left\{ 9, -1 \right\}
 \end{aligned}$$



There is another method we can use to solve a quadratic equation. It involves dividing b by two, squaring the result and adding that value to both sides of the equation. This is known as completing the square.