Essential Question: How can we model the path of an object using a quadratic function?

Do Now:

Consider the quadratic function shown in the table below.

vertex

x	-1	0	1	2	3	4
y	3	9	11	9	3	-7

y values

Which of the following inequalities represents the range of the function?

(a) $y \geq -7$ (b) $y \leq 4$ (c) $y \geq 3$ (d) $y \leq 11$

x values

Which of the following inequalities represents the interval for which the function is increasing?

do not include vertex

(a) $x \geq 1$ (b) $x \leq 1$ (c) $x > 1$ (d) $x < 1$

Parabolas in Real Life (https://www.youtube.com/watch?v=He42k1xRpbQ)

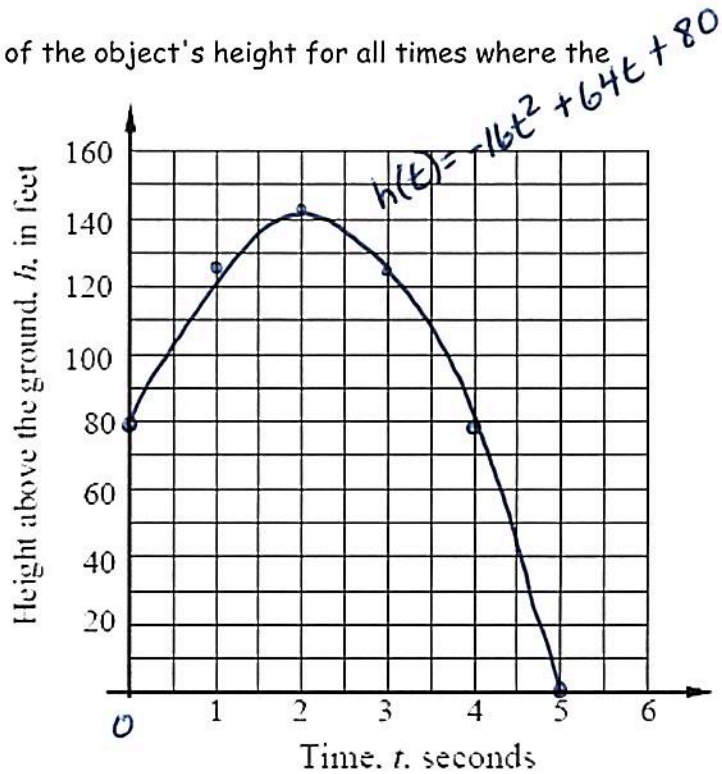
If the question asks...	Then calculate the...	How?
When does an object reach its maximum height?	x value of the vertex	$x = \frac{-b}{2a}$
What is the maximum height of an object?	y value of the vertex	Find $x = \frac{-b}{2a}$ and substitute in equation to find the y value
How long is an object in the air? When does the object hit the ground? (h=0)	roots (zeroes)	set y equal to 0 solve for x
What is the initial height of an object?	y intercept	"c" in the equation or solve for y when x=0

1) The height of an object that is traveling through the air can be modeled by a quadratic function that opens downward. The object is fired upward and its height in feet above the ground is modeled by the equation:

$$h(t) = -16t^2 + 64t + 80$$

a) Create a table of values and draw a graph of the object's height for all times where the object is on or above the ground.

t	$h(t)$
0	80
1	128
2	144
3	128
4	80
5	0



b) What is the maximum height the object reaches (*in feet*)?

y value of vertex

144 feet

c) At what time does the object hit the ground?

h = 0

5 seconds

x values y values

d) State the domain and range of the function.

$0 \le x \le 5$ $0 \le y \le 144$

[0, 5] [0, 144]

e) Over what time interval is the object's height decreasing? (do not include vertex)

x values

$2 < x < 5$

f) Over what time interval is the object's height increasing? (do not include vertex)

x values

$0 < x < 2$

2) A baking soda rocket is fired upwards with an initial speed of 80 feet per second. Its height above the ground, h (in feet), can be modeled using the equation, $\boxed{h(t) = -16t^2 + 80t,}$ where t is the time since the launch (in seconds). At what time does the rocket hit the ground after being launched?

height = 0

$h(t) = 0$

find the roots

$$0 = -16t^2 + 80t$$

$$0 = -16t(t-5)$$

$-16t = 0$ | $t - 5 = 0$

$\cancel{t = 0}$ | $t = 5$

At 5 seconds, the rocket hits the ground.

3) A player hits a baseball into the outfield. The equation $\boxed{h = -0.005x^2 + x + 3}$ models the path of the ball, where h is the height and x is the horizontal distance the ball travels.

(a) What is the maximum height reached by the baseball?

y value of height

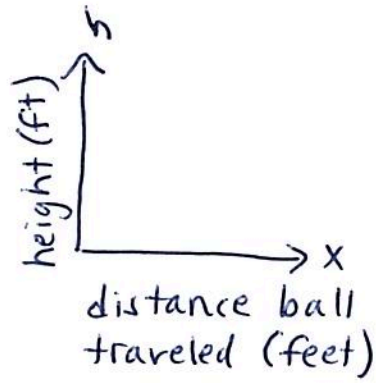
$$x = \frac{-b}{2a}$$

$$x = \frac{-1}{2(-.005)}$$

$$x = 100$$

$$h = -0.005(100)^2 + 100 + 3$$

$$h = 53$$



53 feet is the maximum height

(b) An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it?

$h = 3$

x value when $h = 3$

$$h = -0.005x^2 + x + 3$$

$$3 = -0.005x^2 + x + 3$$

$$0 = -0.005x^2 + x$$

$$0 = x(-0.005x + 1)$$

$x = 0$ | $-0.005x + 1 = 0$

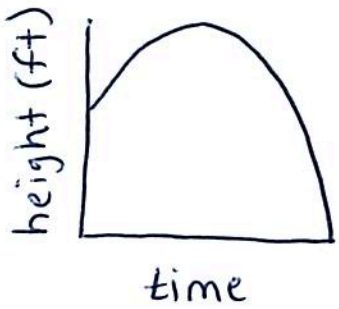
$-0.005x = -1$

$x = 200$

The ball has traveled 200 feet.

4) A manufacturer is testing the durability of an object. He decides to throw the object straight up in the air; the height of the object over time can be modeled by the function $f(t) = -16t^2 + 32t + 48$.

t	$f(t)$
0	48
1	64
2	48
3	0



(a) State a feasible domain for the above stated function. What does the domain of the function represent in this context?

x values

$0 \le x \le 3$

$[0, 3]$

It takes 3 seconds for the object to hit the ground.

(b) State a feasible range for the above stated function. What does the range of the function represent in this context?

y values

$0 \le y \le 64$

$[0, 64]$

The height of the object is at its highest at 64 feet.

(c) At what height does the object get thrown from?

y intercept

48 feet

(d) After how many seconds does the object hit the ground?

$h = 0$

3 seconds

(e) What is the maximum height that the object reaches while in the air? How long does it take for the object to reach this height?

x value of vertex

y value of vertex

64 feet is the maximum height.

It takes one second to reach that height.

The coordinates of the **vertex**, the **y-intercept** and **roots** of a quadratic function help us understand the **parabolic path** of an object.

Turn and Talk

1. A ball is being thrown into the air and follows a path represented by the function $\boxed{f(t) = -4(t+2)^2 + 36}$ where t represents the time since the ball was thrown, in seconds. At what value of t does the ball reach the ground? Justify your response.

$h = 0$

$0 = -4(t+2)^2 + 36$

$0 = -4(t+2)(t+2) + 36$

$0 = -4(t^2 + 4t + 4) + 36$

$0 = -4t^2 - 16t - 16 + 36$

$0 = -4t^2 - 16t + 20$

$0 = -4(t^2 + 4t - 5)$

$0 = -4(t+5)(t-1)$

$t + 5 = 0$	$t - 1 = 0$
$\cancel{t = -5}$	$t = 1$

The ball reaches the ground at one second.

2. A flare is launched from the deck of a lifeboat 4 feet above the water surface. The initial velocity is 80 ft/s. The path of the ball can be modeled using the function $\boxed{h(t) = -16t^2 + 80t + 4}$ where t represents the time since the flare was launched, in seconds. *Algebraically*, determine how many seconds it will take for the flare to be 100 feet above the water's surface.

$h = 100$

$h(t) = -16t^2 + 80t + 4$

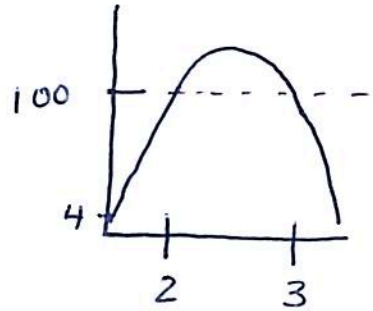
$100 = -16t^2 + 80t + 4$

$0 = -16t^2 + 80t - 96$

$0 = -16(t^2 - 5t + 6)$

$0 = -16(t-3)(t-2)$

$t - 3 = 0$	$t - 2 = 0$
$t = 3$	$t = 2$



Between 2 and 3 seconds The flare is 100 feet above the ground for one second.