Essential Question: What is the quadratic formula and how can it help us solve quadratic equations?

Do Now: Solve the quadratic equation: $x^2 + 7x + 6 = 0$

$$(x+6)(x+1) = 0$$

$x+6=0$	$x+1=0$
$x=-6$	$x=-1$

Up until this point, you have solved quadratic equations in <u>two ways</u>. You have solved quadratic equations in the form of $ax^2 + c = 0$ by finding the **square root** and you have solved quadratic equations in the form of $ax^2 + bx + c = 0$ by **factoring**.

Think about this...
Is there another way to solve a quadratic equation?

The **quadratic formula**, derived from $ax^2 + bx + c = 0$, is a formula that can be used to solve **any** quadratic equation.

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when $a \neq 0$

Let's use the formula to solve the quadratic equation from the Do Now. $x^2 + 7x + 6 = 0$

1) Rewrite as $ax^2 + bx + c = 0$ and identify a, b and c.
2) Write the quadratic formula.
3) Substitute the identified values for a, b and c into the formula.
4) Simplify the expression under the square root symbol ($\sqrt{\ }$). Simplify the denominator.
5) Evaluate the square root (if possible).
6) Write solutions as two equations.
7) Simplify both solutions.

$x^2 + 7x + 6 = 0$

$a = 1$
$b = 7$
$c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{25}}{2}$$

$$x = \frac{-7 \pm 5}{2}$$

$$x = \frac{-7+5}{2}, \frac{-7-5}{2}$$

$$x = -1, -6$$

Think about this....

Can you solve the quadratic equation $x^2 + 2x - 1 = 0$ by factoring?
If you can't factor, what method can you use to solve the equation?

No, no two numbers multiply to -1 and also add to 2.

$x^2 + 2x - 1 = 0$

$a = 1$
$b = 2$
$c = -1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2-4(1)(-1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{8}}{2}$$

$$x = \frac{-2 \pm \sqrt{4}\sqrt{2}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

$x = -1+\sqrt{2}, -1-\sqrt{2}$

What do you notice about the solutions to the quadratic equation? Is there a way to simplify the solutions? They are irrational.

Quick Review: Let's simplify the following radical expressions (*irrational numbers*).

1. $\pm\sqrt{8}$

$\pm\sqrt{4}\sqrt{2}$

$\pm 2\sqrt{2}$

$2\sqrt{2}, -2\sqrt{2}$

2. $\pm\sqrt{27}$

$\pm\sqrt{9}\sqrt{3}$

$\pm 3\sqrt{3}$

$3\sqrt{3}, -3\sqrt{3}$

3. $\pm\sqrt{32}$

$\pm\sqrt{16}\sqrt{2}$

$\pm 4\sqrt{2}$

$4\sqrt{2}, -4\sqrt{2}$

4. $\pm\sqrt{10}$

$\pm\sqrt{10}$

$\sqrt{10}, -\sqrt{10}$

Let's simplify more complicated radical expressions (*irrational numbers*).

5. $\frac{4 \pm 6\sqrt{3}}{2}$

$2 \pm 3\sqrt{3}$

$2+3\sqrt{3}, 2-3\sqrt{3}$

6. $\frac{10 \pm 8\sqrt{7}}{4_2}$

$\frac{5 \pm 4\sqrt{7}}{2}$

$\frac{5+4\sqrt{7}}{2}, \frac{5-4\sqrt{7}}{2}$

7. $\frac{12 \pm \sqrt{24}}{2}$

$\frac{12 \pm \sqrt{4}\sqrt{6}}{2}$

$\frac{12 \pm 2\sqrt{6}}{2}$

$6 \pm \sqrt{6}$

$6+\sqrt{6}, 6-\sqrt{6}$

8. $\frac{8 \pm \sqrt{18}}{2}$

$\frac{8 \pm \sqrt{9}\sqrt{2}}{2}$

$\frac{8 \pm 3\sqrt{2}}{2}$

$\frac{8+3\sqrt{2}}{2}, \frac{8-3\sqrt{2}}{2}$

Now let's take a closer look at the irrational solutions from $x^2 + 2x - 1 = 0$.

$$x = \frac{-2 \pm \sqrt{8}}{2}$$

$$x = \frac{-2 \pm \sqrt{4}\sqrt{2}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

$$x = -1+\sqrt{2},\ -1-\sqrt{2}$$

Solve the quadratic equation below and express your answer in simplest radical form.

$x^2 - 4 = 2x$

$x^2 - 2x - 4 = 0$

$a = 1$
$b = -2$
$c = -4$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{(-2)^2-4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+16}}{2}$$

$$x = \frac{2 \pm \sqrt{20}}{2}$$

$$x = \frac{2 \pm \sqrt{4}\sqrt{5}}{2}$$

$$x = \frac{2 \pm 2\sqrt{5}}{2}$$

$$x = 1 \pm \sqrt{5}$$

$$x = 1+\sqrt{5},\ 1-\sqrt{5}$$

The quadratic formula can be used to solve ***any*** quadratic equation. However, it is most useful when solving quadratic equations that cannot be factored.

JUST ONE more THING!

In how many ways can $x^2 - 4 = 0$ be solved? Name every method that can be used.

$(x-2)(x+2=0$

① DOTS (difference of two squares)

$\sqrt{x^2} = \pm\sqrt{4}$

② square root

③ quadratic formula