Essential Question: How can we represent an exponential function that shows an increase or decrease over time?

Do Now: Read and solve the following word problems.

(a) The radio station Z-100 is sponsoring a contest. The prize begins as a $1000 gift card to Roosevelt Field Mall. Once a day, the disc jockey announces a name, and the person has 15 minutes to call in and claim the prize. If the person does not call within the allotted time, the prize increases by 10% per day. How much will the gift card be worth if no one wins after 3 days?

Number of Days Passed	Mathematical Expression 100% + 10% 110% → 1.1	Prize Money
0	*Initial Value of the Prize*	$1000
1	1000(1.1)	1,100
2	1000(1.1)(1.1)	1,210
3	1000(1.1)(1.1)(1.1)	1,331

(b) Leo purchases a car for $26,499. The car depreciates (loses value) at a rate of 18% annually. What will Leo's car be worth after 3 years?

Number of Years	Mathematical Expression 100% - 18% 82% → .82	Car's Worth
0	*Initial Value*	$26,499
1	26499(.82)	21,729.18
2	26499(.82)(.82)	17,817.93
3	26499(.82)(.82)(.82)	14,610.70

Exponential Growth occurs when a quantity <u>**increases**</u> by the same rate, ***r***, in each unit of time, ***t***.

Exponential Decay occurs when a quantity <u>**decreases**</u> by the same rate, ***r***, in each unit of time, ***t***.

The value of the quantity at any given time can be calculated as a function of the rate and the original amount.

Exponential Growth Model	**Exponential Decay Model**
$y = a\overbrace{(1 + r)}^{\text{growth factor}}{}^{t}$	$y = a\overbrace{(1 - r)}^{\text{decay factor}}{}^{t}$
Time	Time
Final Amount	Final Amount
Initial Amount	Initial Amount
Growth Rate (*expressed as a decimal*)	Decay Rate (*expressed as a decimal*)

Let's look at the ***Do Now...which situation represents exponential growth? exponential decay?***

Exponential Model	**Exponential Model**
$y = 1000(1 + .1)^t$	$y = 26499(1 - .18)^t$
What is the value of the prize money after 3 days have passed? $t = 3$ $y = 1000(1.1)^3$ $y = \$1,331$	What is the value of Leo's car after 3 years? $t = 3$ $y = 26499(.82)^3$ $y = \$14,610.70$

Examples:

initial value

1. A sculpture was valued at $1200 in the year 1990. Since then it has been appreciating at a rate of 8% per year.

increasing

a) Write an exponential function to model this situation.

$a = 1200$

$r = 8\% \rightarrow .08$

$y = 1200(1 + .08)^t$

or

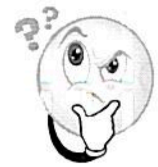
$y = 1200(1.08)^t$

b) Complete the table of values that shows the increase in value over time. *Round to the nearest dollar.*

$y = 1200(1.08)^t$

t (time in yrs)	0	5	10	15	20
y (value in $)	1200	1763	2591	3807	5593

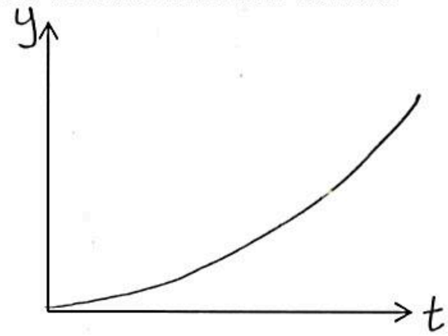
c) Sketch a graph of the function with the indicated window.



Think About This?

What do the window settings mean in the context of the problem?

Window:
x-min: 0
x-max: 40
xscl: 5
y-min: 1,200
y-max: 30,000
yscl: 150



d) How much is the sculpture worth now to the *nearest dollar*?

$$\begin{array}{r} 2019 \\ -\ 1990 \\ \hline 29 \end{array}$$ years → t

$y = 1200(1.08)^t$

$y = 1200(1.08)^{29}$

$y = \boxed{\$11,181}$

2. Mr. Rogers purchased machinery for his farming operation for $175,000. It is expected to depreciate at a rate of 9% per year.
(decay)

a) Write an exponential function to model this situation. What will be the value of the piece of machinery in 10 years?

$a = 175000$

$r = .09$

$y = 175000(1 - .09)^t$

or $y = 175000(.91)^t$

$y = 175000(.91)^{10}$

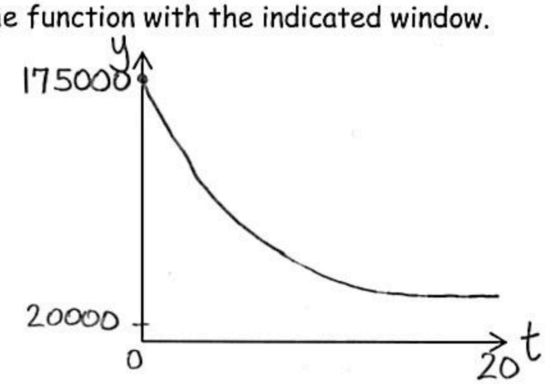
$y = \boxed{\$68,147.82}$

b) Sketch a graph of the function with the indicated window.

decay rate: .09 or 9%

decay factor: .91

Window:
x-min: 0
x-max: 20
xscl: 5
y-min: 20,000
y-max: 175,000
yscl: 10,000



c) Approximately, how many years will it take for the combine to be worth $50,000?

$50000 = 175000(.91)^t$

t	y
12	56433
13	51354
14	46732

Between 13 and 14 years

Is it Exponential Growth or Decay?

3. Maria's parents invested \$14,000 in a CD account earning 6% per year compounded annually. How much money will there be in the account after 10 years?

$a = 14000$

$r = .06$

$t = 10$

$y = 14000(1+.06)^{10}$

or

$y = 14\,000(1.06)^{10}$

$y = \$25,071.87$

4. In 2000, 2200 students attended Polaris High School. The enrollment has since been declining 2% annually. If this trend continues, how many students will be enrolled in 2019?

$a = 2200$

$r = .02$

$t = 19 \quad (2019-2000)$

$y = 2200(1-.02)^{19}$

or

$y = 2200(.98)^{19}$

$y = 1498.711\ldots$

$\boxed{\text{1498 students}}$

5. Ms. Arnold received a job as a teacher with a starting salary of \$55,000. According to her contract, she will receive a 1.5% increase in her salary every year. Write an exponential function that can be used to find S, Ms. Arnold's salary after t years. How many years will it take for Ms. Arnold to reach a minimum salary of \$60,000?

$a = 55000$

$r = .015$

$S = 55000(1+.015)^t$

or

$S = 55000(1.015)^t$

$60000 = 55000(1.015)^t$

t	S
5	59251
6	60139
7	61041

$\boxed{\text{6 years}}$

TAKE AWAY

If a relationship grows over time, it can be represented by an **Exponential Growth** model, $y = a(1+r)^t$, where **1 + *r*** represents the growth factor between successive function values when ***t*** increases by 1.

If a relationship decreases over time, it can be represented by an **Exponential Decay** model, $y = a(1-r)^t$, where **1 - *r*** represents the decay factor between successive function values when ***t*** increases by 1.