Essential Question: How can we determine the rate of change of an exponential function?

Do Now: Consider the exponential function: $f(x) = 8(2)^x$

a) Evaluate $f(3)$.

$f(3) = 8(2)^3$

$f(3) = 64$

or in calculator

$Y = 8(2)^x$

x	y
3	64

Finding x algebraically

$f(x) = 1024$

$1024 = 8(2)^x$

$128 = 2^x$

$2^7 = 2^x \qquad x = 7$

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b) Find the value of x if $f(x) = 1024$.

in calculator $Y = 8(2)^x$
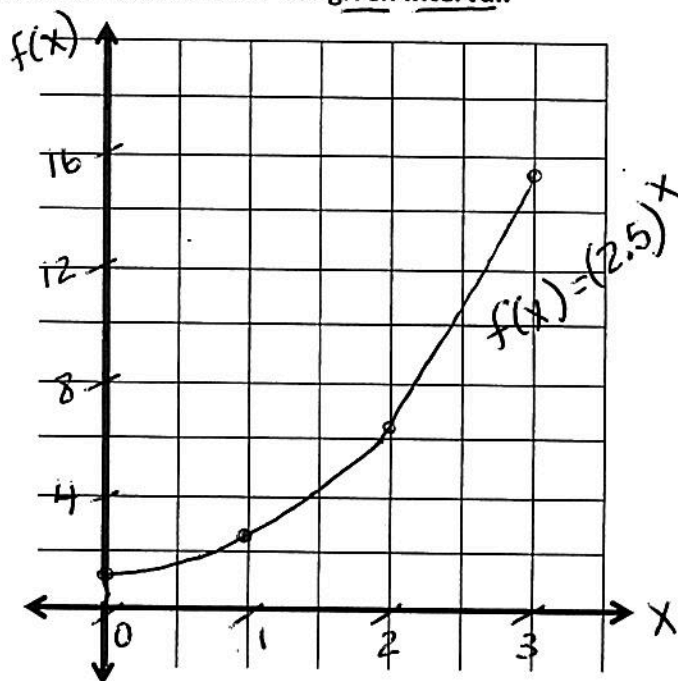
x	y
7	1024

Let's take a closer look at exponential functions.

no arrows!

Make a table of values and graph the following exponential function over the given interval.

1. Graph $f(x) = (2.5)^x$ over the interval $0 \le x \le 3$

x	f(x)
0	$(2.5)^0 = 1$
1	2.5
2	6.25
3	15.625

What is the average rate of change of this function over the given interval?

Interval begins at: (0 , 1)

Interval ends at: (3 , 15.625)

Average Rate of Change: $\dfrac{\Delta y}{\Delta x}$

$\dfrac{15.625 - 1}{3 - 0}$

$= \dfrac{14.625}{3}$

$= 4.875$

Don't FORGET! All **linear** functions have a *constant rate of change*.

In an **exponential** function, an *average rate of change* can be calculated over a specified interval.

2. Consider the exponential function f(x) = 10(2)^x.

   a) Find the value of f(0). What is the significance of this value?

   $f(0) = 10(2)^0$
   $= 10(1) = 10$

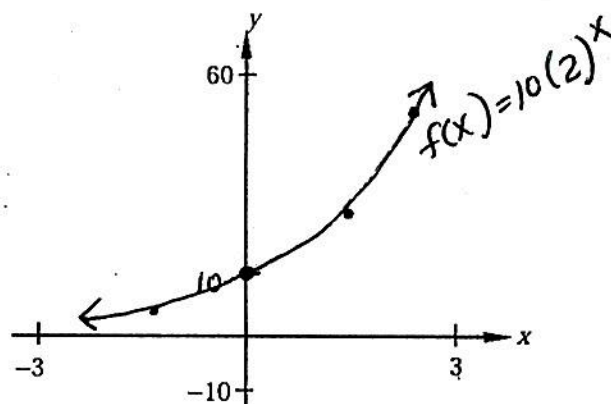   That is the y-intercept of the function.

   b) Is this an increasing or decreasing exponential function? How do you know?

   increasing   the base is greater than 1

   2 > 1

   c) Using your calculator, sketch a graph of this function on the axes shown below. Use the window indicated. Mark the y-intercept.

   y min   −10
   y max   60

   x min   −3
   x max   3

X	Y
−2	2.5
−1	5
0	10
1	20
2	40
3	80

   $f(x) = 10(2)^x$

   d) What is the function's average rate of change over the interval −1 ≤ x ≤ 2?

   $f(-1) = 10(2)^{-1}$
   $= 10\left(\tfrac{1}{2}\right)$
   $= 5$
   (−1, 5)

   $f(2) = 10(2)^2$
   $= 10(4)$
   $= 40$
   (2, 40)

   $\dfrac{\Delta y}{\Delta x} = \dfrac{40-5}{2-(-1)}$
   $= \dfrac{35}{3}$
   $= 11.\overline{6}$

   e) Is this rate of change greater than or less than that of the linear function g(x) = 10x + 7? Explain.

   This rate of change is greater than the R.O.C. of the linear function

   $11.\overline{6} > 10$

   constant rate of change is the slope   10

   TAKE AWAY

   Exponential functions are curved lines that either increase or decrease rapidly. We can determine an *average* ___rate of change___ of a specific part of an exponential function by using the two points that mark the beginning and end of the ___interval___. Use these two points and $\dfrac{\Delta y}{\Delta x}$ to calculate the average rate of change.