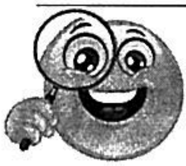
Essential Questions: What is a recursive rule? How do we use recursive formulas to find the terms in a sequence?

Do Now: Create a sequence of numbers using the following information below.

Write the sequence here: 2, 7, 32, 157

1) The first number of the sequence is 2.
2) To find the second number of the sequence, take the first number, multiply it by 5 then subtract 3.
3) To find the third number of the sequence, take the second number, multiply it by 5 then subtract 3.
4) To find the fourth number of the sequence, take the third number, multiply it by 5 then subtract 3.



Let's take a closer look at the sequence from the Do Now.

2, 7, 32, 157

1) Is the sequence arithmetic? No, there is not a constant difference
2) Is the sequence geometric? No, there is not a constant ratio
3) Does the sequence follow a pattern? Yes, multiply by 5, then subtract 3

The sequence from the Do Now can be defined recursively.

Using a Recursive Rule to Generate a Sequence

A **recursive** rule for a sequence defines the **nth** term by relating it to one or more previous terms.

A **recursive formula** will help you find the **next term** in a sequence. Each term is found by doing something (+, -, x, ÷) to the **previous term(s)**.

A **recursive formula** is written with **two parts**:

- a statement of the **starting term**
- a statement of the **formula** used to arrive at the next term

Let's define the sequence from the Do Now recursively.

The **first term** is 2. The **nth** term equals 5 times the **previous term** minus 3.

$a_1 = 2$ $a_n = 5a_{n-1} - 3$

Find the first four terms of each sequence given the recursive rule.

1) $a_1 = 6;\ a_n = 2a_{n-1} + 1$

n		a_n
1	a_1	6
2	$a_2 = 2a_1 + 1$ $= 2(6)+1$	13
3	$a_3 = 2a_2 + 1$ $= 2(13)+1$	27
4	$a_4 = 2a_3 + 1$ $= 2(27)+1$	55

2) $a_1 = -12;\ a_n = \frac{1}{2}a_{n-1} - 4$

n		a_n
1	$a_1 = -12$	-12
2	$a_2 = \frac{1}{2}a_1 - 4$ $= \frac{1}{2}(-12) - 4$	-10
3	$a_3 = \frac{1}{2}a_2 - 4$ $= \frac{1}{2}(-10) - 4$	-9
4	$a_4 = \frac{1}{2}a_3 - 4$ $= \frac{1}{2}(-9) - 4$	-8.5

3) $f(1) = \frac{1}{2};\ f(n) = -4f(n-1) + 6$

n		f(n)
1	$f(1) = \frac{1}{2}$	$\frac{1}{2}$
2	$f(2) = -4f(1) + 6$ $= -4\left(\frac{1}{2}\right) + 6$	4
3	$f(3) = -4f(2) + 6$ $= -4(4) + 6$	-10
4	$f(4) = -4f(3) + 6$ $= -4(-10) + 6$	46

- Sequences defined **recursively** use the *previous term* to find the next term of the sequence.
- The symbols $\mathbf{a_{n-1}}$ and $\mathbf{f(n-1)}$ represent the *previous term* in the sequence.

More Recursive Rules

4) If a sequence is defined recursively by f(1) = 10 and f(n) = -f(n-1) + 3 then find f(5).

multiply by -1 add 3

$$f(1) = 10$$
$$f(2) = -f(1) + 3 = -10 + 3 = -7$$
$$f(3) = -f(2) + 3 = -(-7) + 3 = 10$$
$$f(4) = -f(3) + 3 = -(10) + 3 = -7$$
$$f(5) = -f(4) + 3 = -(-7) + 3 = 10$$

5) A sequence is defined recursively by adding 4 to twice the value of the previous term. Write a recursive rule for this sequence if the first term is 15.

$$a_n = 2a_{n-1} + 4, \quad a_1 = 15$$

The TAKE AWAY

When creating a sequence using a **recursive rule**, it is important to understand that each term of the sequence is found using the ___previous___ term. If the nth term of the sequence is denoted by a_n then the previous term is ___a_{n-1}___.

PIPS CORNER

One of the most famous sequences in mathematics is The Fibonacci Sequence. It is defined recursively as follows: $a_n = a_{n-1} + a_{n-2}$; $a_0 = 1$ and $a_1 = 1$. Using this rule, find the first five terms of the sequence.

$$a_0 = 1$$
$$a_1 = 1$$
$$a_2 = a_1 + a_0 = 1 + 1 = 2$$
$$a_3 = a_2 + a_1 = 2 + 1 = 3$$
$$a_4 = a_3 + a_2 = 3 + 2 = 5$$
$$a_5 = a_4 + a_3 = 3 + 5 = 8$$