

Essential Question: How can we distinguish between arithmetic and geometric sequences?

Do Now:

- i) Identify each sequence as arithmetic, geometric or neither.  
 ii) If arithmetic, identify the common difference. If geometric, identify the common ratio.

A. 12, 18, 27, 40.5, ... geometric 1.5

B. -123, -137, -151, -165, ... arithmetic  $d = -14$

C. 3, 7, 15, 31, ... neither

D.  $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \dots$  geometric  $r = \frac{1}{4}$

STOP HERE



1. For letters A. and B. above, write an equation that can be used to find the  $n$ th term of the sequence.

$$a_1 = 12$$

$$a_1 = -123$$

12, 18, 27, 40.5, ...  $r = 1.5$

-123, -137, -151, -165, ...  $d = -14$

A.  $a_n = 12(1.5)^{n-1}$

B.  $a_n = -123 - 14(n-1)$

2. Using your equation, find the 10<sup>th</sup> term in each sequence.

$$\begin{aligned} a_{10} &= 12(1.5)^{10-1} \\ &= 12(1.5)^9 \\ &= 461.3203125 \end{aligned}$$

$$\begin{aligned} a_{10} &= -123 - 14(10-1) \\ a_{10} &= -123 - 14(9) \\ a_{10} &= -249 \end{aligned}$$

3. Katie works at the local pet shop. For a single litter of kittens, she puts out 17 ounces of wet food. For 2 litters she puts out 34 ounces of wet food and for 3 litters, she puts out 51 ounces of wet food. She continues this pattern for  $n$  litters.

a) Write an equation that can be used to find the number of ounces of wet food,  $a_n$ .

litters  $\downarrow$  Katie will put out for  $n$  litters of kittens.

n	1	2	3
$a_n$	17	34	51

$$a_1 = 17$$

$$d = 17$$

$$a_n = 17 + 17(n-1)$$

ounces of food  $\uparrow$

b) How much wet food will Katie put out if there are 8 litters of kittens in the store?

$$a_8 = 17 + 17(8-1)$$

$$n = 8$$

$$a_8 = 136$$

136 ounces of wet food

4. A soup kitchen makes 16 gallons of soup every two weeks. Each day they serve 25% of the soup that remains from the previous day. The table below shows how much soup,  $f(n)$ , remains after  $n$  days.

# of days $\downarrow$ n	1	2	3
f(n)	12	9	6.75

soup remaining (gallons)  $\uparrow$

a) Write an equation that can be used to find the number of gallons of soup remaining after  $n$  days.

$$f(1) = 12$$

$$r = .75$$

$$f(n) = 12(.75)^{n-1}$$

b) How many gallons of soup remain after the 12<sup>th</sup> day? Round your answer to the nearest tenth.

$$n = 12$$

$$f(12) = 12(.75)^{12-1}$$

$$f(12) = 12(.75)^{11}$$

$$f(12) = .5$$

A half gallon of soup remains after the 12th day.

c) On what day is there about 2 gallons of soup left?

$$n \quad f(n)$$

$$2 = 12(.75)^{n-1}$$

First, put this in the calculator  
 $y = 12(.75)^{x-1}$

Then, look at the y column in the table of values. when is it closest to 2? The 7<sup>th</sup> day

5. Write an explicit rule for an arithmetic sequence if  $a_6 = 8$  and  $a_{10} = 40$ .

8				40
$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$

  

6	7	8	9	10	11
8				40	

To find the common difference, subtract the numbers and divide by the number of places. Find the R.O.C.

$$\frac{\Delta y}{\Delta x} = \frac{40 - 8}{10 - 6} \rightarrow \frac{32}{4} \rightarrow 8$$

$d = 8$

To find the first term, it is 5 places away from the 6th term.

$$a_1 = a_6 - 5(8)$$

$$a_1 = 8 - 40$$

$$a_1 = -32$$

Find the 20th term

$$a_n = -32 + 8(n-1)$$

$$a_{20} = -32 + 8(20-1)$$

$$a_{20} = 120$$

6. Write an explicit rule for a geometric sequence if  $a_3 = 10$  and  $r = \frac{1}{2}$ .

$a$	1	2	3	4
$a_n$	40	20	10	

To find the previous term, divide by the ratio

$$10 \div \frac{1}{2} = 20$$

$$20 \div \frac{1}{2} = 40$$

The explicit rule finds the value of any term

$$a_n = 40(.5)^{n-1}$$

## The TAKEAWAY

If a sequence of numbers is **arithmetic**, the pattern will display a common difference between consecutive terms. An explicit formula  $a_n = \underline{a_1 + d(n-1)}$  can be used to find the  $n$ th term of the sequence.

If a sequence of numbers is **geometric**, the pattern will display a common ratio between consecutive terms. An explicit formula  $a_n = \underline{a_1(r)^{n-1}}$  can be used to find the  $n$ th term of the sequence.