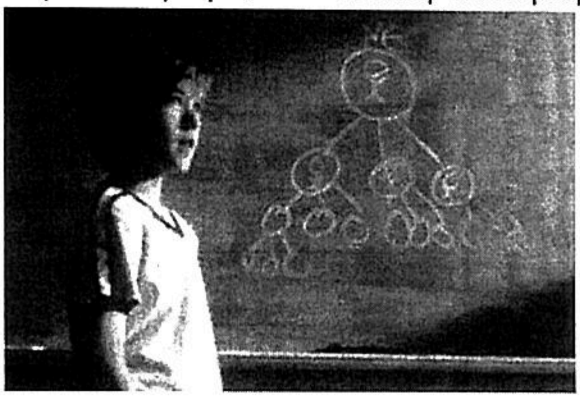
Essential Questions: What is a geometric sequence? How do we define geometric sequences explicitly?

Do Now: In the movie "Pay it Forward" the main character, a young boy, determines that he can make a significant difference in the world by creating a chain of events. During the movie he helps three people, who each help three people and so on.



(a) How many people's lives would be affected in the 6th round of this pattern?

1, 3, 9, 27, 81, 243

(b) Identify the pattern in this sequence of numbers.

(https://www.youtube.com/watch?v=KxB43PxasGA)

What is a Geometric Sequence?

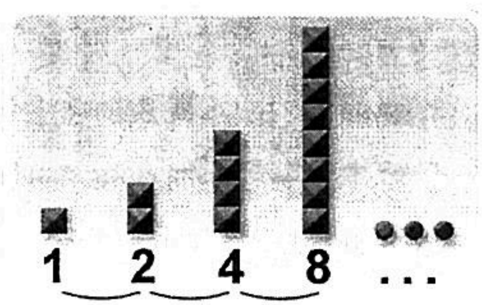
If a sequence of values follows a pattern of **multiplying** a fixed amount (not zero) to arrive at the next term, it is referred to as a **geometric sequence**. In a geometric sequence, the ratio of successive terms is called the **common ratio (r)**.

To find the common ratio: Divide any term by the previous term. $8 \div 4$

- The common ratio in this example is 2.

To find the next term: Multiply the previous term by the common ratio. 8×2

- The next term in this example is 16.



Let's take a look at some sequences...is there a common ratio? If so, find the next term in the sequence.

(1) 1, -2, 4, -8, ...

$4 \div -2 = -2$

$-8 \div 4 = -2$

yes

$r = -2$

(2) 3, 6, 10, 15, ...

$6 \div 3 = 2$

$15 \div 10 = 1.5$

No

(3) 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...

$\frac{1}{2} \div 1 = \frac{1}{2}$

$\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$

yes, $r = \frac{1}{2}$

Writing Geometric Sequences as Functions

You can use the first term and the common ratio to write a function rule that describes a geometric sequence. Assume the first term is 4 and the common ratio is 3.

$a_1 =$ 4 $r =$ 3

Term # n	Term a_n	Written in terms of a_1 and r	Term
1	a_1	a_1	4
2	a_2	$a_1 \cdot r$	$4 \cdot 3 = 12$
3	a_3	$a_1 \cdot r \cdot r \dashrightarrow a_1 \cdot r^2$	$4(3)^2 = 36$
4	a_4	$a_1 \cdot r \cdot r \cdot r \dashrightarrow a_1 \cdot r^3$	$4(3)^3 = 108$
n	a_n	$a_1 \cdot r^{n-1}$	$4(3)^{n-1}$

The **Explicit Formula** to find the *n*th term of a **geometric sequence**:

Subscript Notation $a_n = a_1 \cdot r^{n-1}$

Function Notation $a(n) = a(1) \cdot r^{n-1}$

(4) Given the following geometric sequence: 1, 4, 16, 64, ...

a) Define the sequence explicitly.

$a_1 =$ 1 $r =$ 4

$a(n) = 1 \cdot 4^{n-1}$

b) Find the 11th term. $n =$ 11

$a(n) = 1 \cdot 4^{n-1}$

$a(11) = 1 \cdot 4^{11-1}$

$= 1 \cdot 4^{10}$

$= 1{,}048{,}576$

(5) Given the following geometric sequence: 128, 32, 8, 2, 0.5, ...

a) Write an equation to find the nth term.

$a_1 =$ 128 $r =$ $\frac{1}{4}$

$$a_n = 128 \cdot \left(\frac{1}{4}\right)^{n-1}$$

b) Find the 8th term. $n =$ 8

$$a_n = 128 \cdot \left(\frac{1}{4}\right)^{8-1}$$

$$= 128 \cdot \left(\frac{1}{4}\right)^{7}$$

$$= .0078125$$

(6) Given the following geometric sequence:

n	1	2	3	4
a_n	$\frac{2}{3}$	-2	6	-18

a) Write an equation to find the nth term.

Find a_1 and ratio

$a_1 = \frac{2}{3}$

$-2 \div \frac{2}{3} = -3$

$-18 \div 6 = -3$

$$a_1 = \frac{2}{3} \cdot (-3)^{n-1}$$

b) Find the 7th term.

$n = 7$

$$a_7 = \frac{2}{3} \cdot (-3)^{7-1}$$

$$= \frac{2}{3}(-3)^6$$

$$= \frac{2}{3}(729)$$

$$= 486$$

The TAKE AWAY

- The ratio of successive terms in a geometric sequence is called the common ratio.
- The explicit formula for a geometric sequence allows you to find the nth term of the sequence by substituting the values of a_1 (first term) and r (common ratio) in the equation $a_n =$ $a_1 \cdot r^{n-1}$.