

Essential Question: How can we model situations using an arithmetic explicit formula?

Do Now:

During halftime at a football game, a high school marching band marches onto the field to perform a routine. There is one performer in the first row, four performers in the second row, and seven performers in the third row. This pattern continues for n rows.

- (a) Create a sequence that represents the number of performers in each row.

$$1, 4, 7, 10, \dots$$



- (b) Write an equation that can be used to find the number of performers in the n th row.

$$a(1) = 1$$

$$d = 3$$

$$a(n) = 1 + 3(n-1)$$

- (c) How many performers are in the 6th row?

$$n = 6$$

$$a(6) = 1 + 3(6-1)$$

$$= 1 + 3(5) \rightarrow 16$$

16 performers
are in the
6th row

Problem Solving and Arithmetic Explicit Formulas

- 1) The first row of a theater has 15 seats in it. Each subsequent row has 4 more seats than the previous row.

$$a(1) = 15$$

$$+4$$

- (a) Write an equation that can be used to find the number of seats in the n th row.

$$a(1) = 15$$

$$d = 4$$

$$a(n) = 15 + 4(n-1)$$

- (b) Find the number of seats in the fifth row.

$$n = 5$$

$$a(5) = 15 + 4(5-1)$$

$$= 15 + 4(4)$$

$$= 31$$

31 seats

- (c) If the last row has 83 seats, how many rows are in the theater?

$$a(n)$$

$$a(n) = 15 + 4(n-1)$$

$$83 = 15 + 4(n-1)$$

$$\begin{array}{r} 83 \\ -15 \\ \hline 68 \end{array} \quad \begin{array}{r} 15 \\ -15 \\ \hline 0 \end{array}$$

$$\frac{68}{4} = \frac{4(n-1)}{4}$$

$$17 = n-1$$

$$n = 18$$

18 rows

- 2) The height (in feet) of the water in a tank each hour after opening its drain can be estimated by the sequence displayed in the table below.

Hours (n)	1	2	3	4
Water Height (a_n)	18	15	12	9

- (a) Write an explicit formula that represents the arithmetic sequence.

$$a_n = 18$$

$$d = -3$$

$$a_n = 18 - 3(n-1)$$

- (b) Find the seventh term. What does this value represent in the context of the situation?

$$n = 7$$

$$a_7 = 18 - 3(7-1)$$

$$= 18 - 3(6)$$

$$= 18 - 18$$

$$= 0$$

In 7 hours,
the tank has
no water left.

- (c) Would the eighth term apply in this situation? Explain.

No, it would create a negative term
in the sequence

- (d) Simplify the explicit formula from part (a). Compare and contrast both formulas.

Simplify in the space below ↓	Original Formula $a_n = 18 - 3(n-1)$	Simplified Formula $21 - 3n$
$a_n = 18 - 3(n-1)$ $= 18 - 3n + 3$ $= 21 - 3n$	<p>comparison (same)</p> <ul style="list-style-type: none"> n is used -3 is the constant change <p>contrast</p> <p>starting point is different</p>	

- (e) What was the height of the water in the tank before the drain was opened?

21 feet (look at y intercept
of simplified formula)

- 3) Caitlin is given a Starbucks card worth \$50. After she purchases a latte, the card's value is \$45.50. After she purchases a second latte, its value is \$41.

(a) Assuming the pattern continues, write an equation $A(n)$, the amount of money on the Starbucks card after n lattes are purchased. Complete the table below to help you.

of
lattes
worth
of
card

n	0	1	2	3	4	5	6
$A(n)$	50	45.50	41	36.50	32	27.50	23

$$A(n) = 45.50 - 4.5(n-1)$$

$$A(n) = 50 - 4.5n$$

- (b) Caitlin buys a latte every Sunday. How many weeks in a row can she afford to buy a latte, using her Starbucks card only?

$$0 = 45.50 - 4.5(n-1)$$

$$\begin{array}{r} -45.50 = -4.5(n-1) \\ \hline -4.5 \quad -4.5 \end{array}$$

$$10.\bar{1} = n-1$$

$$11.\bar{1} = n$$

she can afford to
buy a latte
11 weeks in a row.

$$A(n) = 50 - 4.5n$$

$$0 = 50 - 4.5n$$

$$\begin{array}{r} -50 = -4.5n \\ \hline -4.5 \quad -4.5 \end{array}$$

$$n = 11.\bar{1}$$

TAKEAWAY

Arithmetic sequences represent linear relationships. These sequences can be defined using the formula $a(n) = a(1) + d(n-1)$. In the formula, a_1 represents the first term and d represents the common difference. These sequences can also be defined using the linear function rule $a_n = mn + b$ where m represents the common difference and b represents the term before the first.