9-3

**Essential Question: How do we solve real-world problems using a system of linear inequalities?**

**Do Now:**

Sergio is building a garden. He wants the length of the garden to be at least 30 feet and the perimeter of the garden to be no more than 100 feet.

Write a system of linear inequalities that represents the situation described. Let $x$ represent the length of the garden and let $y$ represent the width.

$x$: length (feet)

$y$: width (feet)

$x \geq 30$

$2x + 2y \leq 100$

## MODELING WITH SYSTEMS OF INEQUALITIES

*There are many situations that arise in business and engineering that necessitate the use of a system of linear inequalities. The region in the coordinate plane that solves the system represents all of the possible solutions to the problem.*

**Example 1:**

Sergio is building a garden. He wants the length of the garden to be at least 30 feet and the perimeter of the garden to be no more than 100 feet. Let $x$ represent the length of the garden and let $y$ represent the width.

(a) Using the system you created in the Do Now, determine all the possible dimensions of the garden by graphing the system.

$x \geq 30$

| x | y |
|---|---|
| 30 | 0 |
| 30 | 10 |

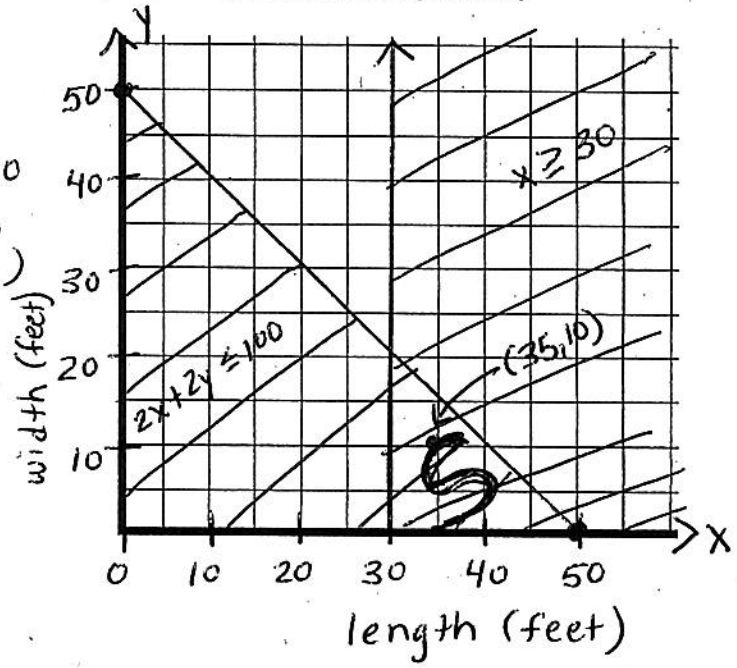
$2x + 2y \leq 100$

$2y \leq -2x + 100$

$y \leq -x + 50$

boundary line → (graph $y = -x + 50$)

| x | y |
|---|---|
| 0 | 50 |
| 50 | 0 |




(b) Is a length of 35 feet and a width of 10 feet a possible combination? How do you know?

look at point on graph

Yes, the point (35, 10) is in the solution set of the graph.

or

substitute in both inequalities

$x \geq 30$

$35 \geq 30$ ✓

$2x + 2y \leq 100$

$2(35) + 2(10) \leq 100$

$70 + 20 \leq 100$

$90 \leq 100$ ✓

(c) State another set of dimensions possible for the garden.

Answers will vary

ex. length of 40 feet, width of 5 feet

**Example 2:**

Paul works $x$ hours a week at a bagel shop that pays $6 an hour. He has also accepted a job that pays $12 an hour mowing lawns for $y$ hours a week. He will work both jobs. Paul wants to earn at least $120 a week, but due to school commitments, he must work less than 30 hours a week.

$< 30$ $\quad \geq 120$

(a) Write a system of inequalities that describes the situation. Graph the system.

x : hours at bagel shop

y : hours mowing lawns

$x + y < 30$ (dashed line)

$y < -x + 30$

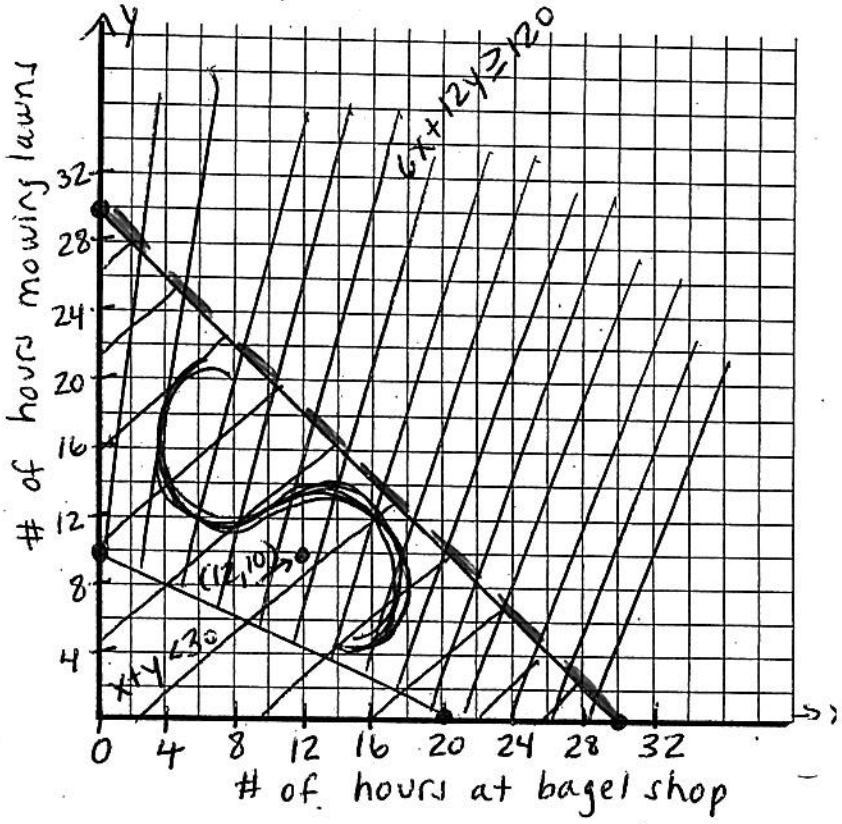
| x | y |
|---|---|
| 0 | 30 |
| 30 | 0 |

$6x + 12y \geq 120$

$12y \geq -6x + 120$

$y \geq -\frac{1}{2}x + 10$

| x | y |
|---|---|
| 0 | 10 |
| 20 | 0 |




(b) Determine and state one combination of hours that will allow Paul to earn at least $120 per week while working less than 30 hours.

Answers will vary

(12, 10)

↑ hours at bagel shop ↑ hours mowing lawn

check:

$x + y < 30$

$12 + 10 < 30$

$22 < 30$ ✓

$6x + 12y \geq 120$

$6(12) + 12(10) \geq 120$

$72 + 120 \geq 120$

$192 \geq 120$ ✓


Systems of Linear Inequalities help us develop solution sets to different types of problems. When developing a system, use two variables to represent two different quantities. Write two inequalities that describe the situation. The solution set to the problem is represented by the ordered pairs shown in the region where both graphs intersect.

**Example 3:** $\leq 500$

The Royal Crown Players of Roslyn High School are raising money for their club by putting on a production of The Music Man. They have 500 seats in the auditorium. They are selling student tickets for $5 each and non-student tickets for $10 each. They must sell at least $2000 worth of tickets to cover their expenses. $\geq 2000$

(a) If $x$ represents the number of student tickets sold and $y$ represents the number of non-student tickets sold, write a system of inequalities that can be used to model this situation. Graph the system.

x: # of student tickets sold
y: # of non-student tickets sold

$x + y \leq 500$

$y \leq -x + 500$

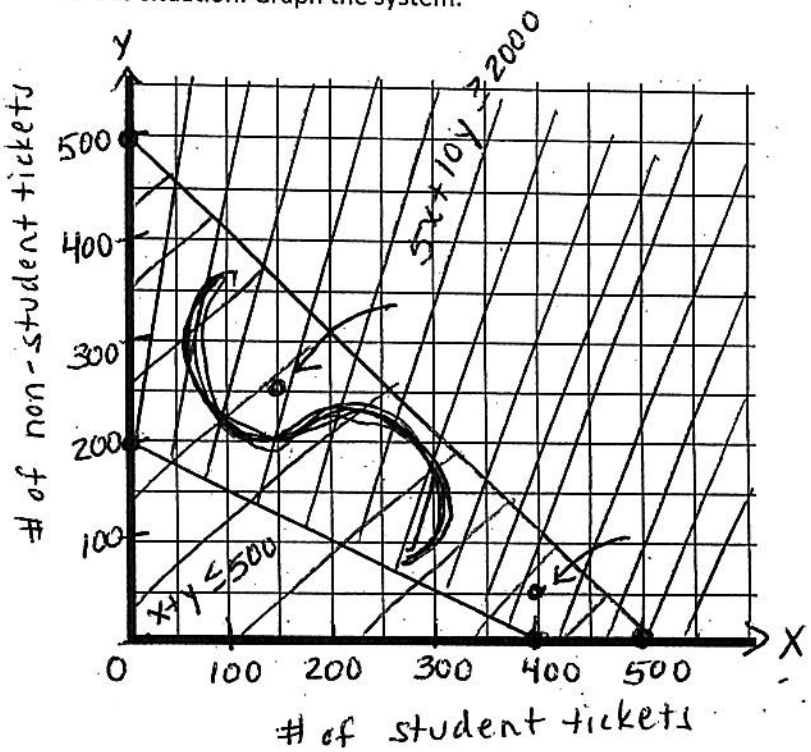
| x | y |
|---|---|
| 0 | 500 |
| 500 | 0 |

$5x + 10y \geq 2000$

$10y \geq -5x + 2000$

$y \geq -\frac{1}{2}x + 200$

| x | y |
|---|---|
| 0 | 200 |
| 400 | 0 |




(b) List two possible combinations of student and non-student tickets that must be sold to cover the club's expenses.

Answers will vary

400 student tickets and 50 non-student tickets

check

$x + y \leq 500$ : $400 + 50 \leq 500$, $450 \leq 500$

$5x + 10y \geq 2000$ : $5(400) + 10(50) \geq 2000$, $3000 \geq 2000$

150 student tickets and 250 non-student tickets

$x + y \leq 500$ : $150 + 250 \leq 500$, $400 \leq 500$

$5x + 10y \geq 2000$ : $5(150) + 10(250) \geq 2000$, $750 + 2500 \geq 2000$, $3250 \geq 2000$

(c) Will the club cover their expenses if they sell 150 student tickets and 100 non-student tickets? Justify your response.

must make at least $2000

$5x + 10y \geq 2000$

$5(150) + 10(100) \geq 2000$

$750 + 1000 \geq 2000$

$1750 \geq 2000$

X

No, they will not cover their expenses because $1750 is less than $2000

## IT'S YOUR TURN NOW!

Karen likes her job as a babysitter, but it pays only $5 per hour. She has been offered a job as a tutor that pays $10 per hour. Because of school work, her parents only allow her to work a maximum of 20 hours per week. How many hours can Karen tutor and babysit if she wants to earn at least $100 per week? $\leq 20$ $\geq 100$

(a) Write a system of inequalities that can be used to answer the question. Use $x$ to represent the number of hours Karen babysits and $y$ to represent the number of hours Karen tutors.

x: # of hours of babysitting
y: # of hours of tutoring

$x + y \leq 20 \longrightarrow y \leq -x + 20$

| x | y |
|---|---|
| 0 | 20 |
| 20 | 0 |

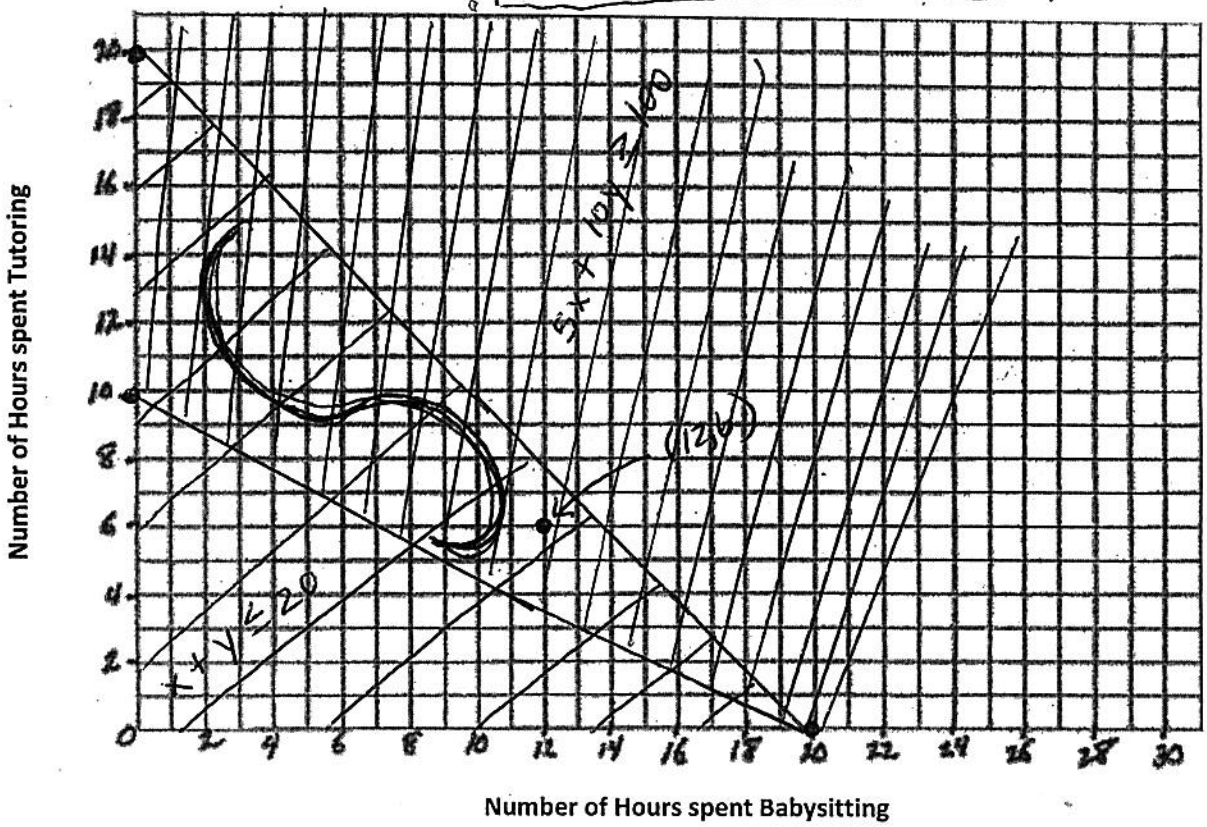
$5x + 10y \geq 100$

$10y \geq -5x + 100$

$y \geq -\frac{1}{2}x + 10$

| x | y |
|---|---|
| 0 | 10 |
| 20 | 0 |

(b) Graph the system.




(c) Determine and state one solution that would allow Karen to work a maximum of 20 hours while making at least $100 in one week. *Explain* your solution in the context of the situation.

(12, 6)

Answers will vary

when Karen babysits for 12 hours and tutors for 6 hours, she does not work more than 20 hours and she makes at least $100 for the week

proof →

$x + y \leq 20$
$12 + 6 \leq 20$
$18 \leq 20$

$5x + 10y \geq 100$
$5(12) + 10(6) \geq 100$
$120 \geq 100$