ALGEBRA 1 MIDTERM PRACTICE PROBLEM SET

1) The Real Number System and Properties

1. Which number is irrational?

(1) $\sqrt[3]{27}$ (3) $\sqrt{3}$

(2) 3.14 (4) $1.\overline{3}$

(3) $\sqrt{3} \approx$ **1.7320508…**

(1) $\sqrt[3]{27} = 3$ *rational*
(2) 3.14 is a terminating decimal *rational*
(4) $1.\overline{3}$ is a repeating decimal *rational*

2. What is the best sequence of words to describe the set of numbers below?

$$\{1\frac{3}{5},\ -5,\ 0.3\overline{2},\ \sqrt[3]{64},\ \sqrt{60},\ 17\}$$

(1) rational, integer, irrational, natural, irrational, whole
(2) real, rational, rational, irrational, irrational, natural
(3) rational, integer, rational, whole, irrational, natural
(4) real, natural, rational, integer, irrational, whole

(3) rational, integer, rational, whole, irrational, natural

(1) $0.3\overline{2}$ *rational*
(2) $\sqrt[3]{64} = 4$ *rational*
(4) -5 is not a natural # (*it's an integer*)

3. Which expression represents $\sqrt{48}$ in simplest radical form?

(1) $2\sqrt{12}$ (3) $4\sqrt{3}$

(2) $8\sqrt{3}$ (4) $4\sqrt{12}$

$\sqrt{48}$
$\sqrt{16} \bullet \sqrt{3}$
$4\sqrt{3}$

(3) $4\sqrt{3}$
(1) Equivalent but *not simplified*
(2) Not equivalent
(4) Not equivalent

4. **Given:**

A = $\sqrt{3}$ B = $\sqrt{6}$ C = $\sqrt{9}$

Which expression results in a rational number?

(1) A + B

(2) B + C

(3) A^2

(4) BC

(3) $A^2 = (\sqrt{3})(\sqrt{3}) = \sqrt{9} = 3$ *rational*

(1) A + B = $\sqrt{3} + \sqrt{6}$ = **I** + **I** = *irrational*
(2) B + C = $\sqrt{6} + \sqrt{9}$ = **I** + **R** = *irrational*
(4) BC = $(\sqrt{6})(\sqrt{9}) = 3\sqrt{6}$ = *irrational*

5. State **true** or **false** for each statement. Justify false responses with an example.

a) The product of two irrational numbers is always irrational.

False The product can be rational or irrational
$\sqrt{2} \bullet \sqrt{3} = \sqrt{6} \leftarrow$ irrational $\quad\quad \sqrt{2} \bullet \sqrt{8} = \sqrt{16} = 4 \leftarrow$ rational

b) The product of a rational number and an irrational number is always irrational.

False The product is always irrational except when the rational number is 0
$\sqrt{2} \bullet 0 = 0 \leftarrow$ rational

c) The sum of two irrational numbers is always irrational.

False The sum can be rational or irrational
$\sqrt{3} + \sqrt{3} = 2\sqrt{3} \leftarrow$ irrational $\quad\quad -\sqrt{3} + \sqrt{3} = 0 \leftarrow$ rational (zero pair)

d) The sum or product of a rational and irrational number is always irrational.

False The sum is always irrational but the product can be rational (*see letter b*) When an irrational number is multiplied by zero (a rational number), the result is zero which is rational.

6. A part of Jennifer's work to solve the equation $3(5x^2 - 4) = 12x^2 - x$ is shown below.

Given: $3(5x^2 - 4) = 12x^2 - x$
Step 1: $15x^2 - 12 = 12x^2 - x$

Which property justifies her first step?

(1) identity property of multiplication
(2) multiplication property of equality
(3) commutative property of multiplication
(4) distributive property of multiplication over subtraction

2) Polynomial Expressions

Perform the indicated operation. Express answers in standard form.

7. $(r^2 + 2r - 1) + (3r^2 - 7r + 2)$ Add

$r^2 + 2r - 1 + 3r^2 - 7r + 2$
$r^2 + 3r^2 + 2r - 7r - 1 + 2$
$4r^2 - 5r + 1$

8. $(7b^3 + b^2 - 3) - (8b^3 - 2b^2 + 5b + 1)$ Subtract

distribute the (-) sign
$7b^3 + b^2 - 3 - 8b^3 + 2b^2 - 5b - 1$
$7b^3 - 8b^3 + b^2 + 2b^2 - 5b - 3 - 1$
$-b^3 + 3b^2 - 5b - 4$

9. $(1 - 2x)(5 + 3x)$

$(1 - 2x)(3x + 5)$
$3x + 5 - 6x^2 - 10x$
$-6x^2 - 7x + 5$

10. $(x + 5)(x^2 - 4x + 9)$

	x^2	$-4x$	9
x	x^3	$-4x^2$	$9x$
5	$5x^2$	$-20x$	45

$x^3 - 4x^2 + 5x^2 + 9x - 20x + 45$

$x^3 + x^2 - 11x + 45$

11. Rewrite the polynomial expression as a trinomial in standard form.

$3x(x + 5) - 7(x + 2)^2$

Follow the order of Operations (PEMDAS)

$3x(x + 5) - 7(x + 2)^2$ ← square the binomial x + 2
$3x(x + 5) - 7[(x + 2)(x + 2)]$
$3x(x + 5) - 7[x^2 + 2x + 2x + 4]$
$3x(x + 5) - 7(x^2 + 4x + 4)$ ← multiply (use distributive property → distribute 3x and -7)
$3x^2 + 15x - 7x^2 - 28x - 28$
$3x^2 + 15x - 7x^2 - 28x - 28$ ← combine like terms
$-4x^2 - 13x - 28$

12. Which expression represents "the sum of 5 and a number subtracted from 17"?

(1) x + 5 - 17
(2) 17 – (x + 5)
(3) 17 – x + 5
(4) 17 + x – 5

(2) 17 – (x + 5)

Subtract from 17 means 17 –
The sum of 5 and a number is x + 5
When subtracting a binomial (more than one term), use ( ).

13. The sum of Scott's age and Greg's age is 33 years. If Greg's age is represented by ***g***, which expression represents Scott's age?

(1) 33 - *g*

(2) *g* - 33

(3) *g* + 33

(4) 33*g*

> **(1) 33 – *g*** Total Sum – Greg's age = Scott's age
>
> Ex: If Greg is 30 years old, then Scott is 3 years old (*33 – 30 = Scott*)

14. The Ambrose family has 3 children, each born two years apart. If the age of the youngest child is represented by **x + 3** ,which expression represents the age of the oldest child?

(1) x + 5

(2) x + 6

(3) x + 7

(4) x + 8

> **(3) x + 7**
>
> Add 2 to x + 3 to find the middle child's age.
> The expression x + 5 represents the age of the middle child.
> Add 2 more and x + 7 represents the age of the oldest child.

15. Katie and Lisa are splitting the cost of a flower arrangement made up of 7 roses and 3 violets.

The expression $\frac{7r+3v}{2}$ is used to find the amount each person will pay.

A. What does the variable ***r*** represent in the expression?

> **r represents the price of one rose**

B. What does **3v** represent in the expression?

> **3v represents the total cost of 3 violets**

C. Using the expression, determine the amount of money each person will spend if red roses sell for $2.25 each and violets sell for $1.75 each.

$$\frac{7r+3v}{2}$$

$$\frac{7(2.25)+3(1.75)}{2}$$

$$\frac{15.75+5.25}{2}=\frac{21}{2}=10.5$$

Each person will pay $10.50

16. The RMS Mathletes have been selected to compete internationally. The club members are holding a school dance in order to raise money for travel expenditures. They are charging students an admission fee and selling bottles of water. They have made a list of expenses and revenue.

a) Using the list, write a *simplified* polynomial expression that represents their profit from the dance if ***s*** students attend and ***b*** bottles of water are purchased.

Revenue	***Expenses***
Water Bottle Sales - $2.50 per bottle	*DJ- $300*
Student Admission - $5.00 per student	*Water Bottle Purchase - $1.00 per bottle*
PFA Donation - $1000	

Profit = Revenue (Income) – Expenses (Cost)

Profit = (2.5b + 5s + 1000) – (300 + 1b) ← distribute the – sign

2.5b + 5s + 1000 – 300 – b

1.5b + 5s + 700

b) Using your expression, calculate the club's profit if 315 students attend the dance and they sell 219 bottles of water.

s = 315 b = 219

Profit = 1.5b + 5s + 700

= 1.5(219) + 5(315) + 700

= 328.5 + 1575 + 700

= 2603.50

The club's profit is $2603.50

3) Equations

17. Which equation *does not* have a solution set equal to 5?

(1) $3(x-4) = \frac{15}{x}$

(3) $\frac{1}{5}x = x - 3$

(2) $2(x+10) = 30$

(4) $0.5x - 2 = 0.5$

> **(3)** $\frac{1}{5}x = x - 3$ $\quad \frac{1}{5}(5) \neq 5 - 3$
>
> $1 \neq 2$
>
> Substitute 5 in for x for each equation. In all the other equations (choices (1), (2) and (4)), the value of x that makes the equation true is 5.

18. What is the solution to the equation **3(x + 4) = 2x + 15 + x** ?

(1) x = 0

(3) the equation has no solution

(2) x = -1

(4) x = all real numbers

> **(3) no solution**
>
> $3(x+4) = 2x + 15 + x$
>
> $3x + 12 = 3x + 15$
>
> $-3x \quad\quad -3x$
>
> $12 \neq 15$

19. If **4xyz = 2**, then what is the value of **x** in terms of **y** and **z**?

(1) $\frac{1}{2yz}$

(3) $2 - 4yz$

(2) $\frac{1}{2}yz$

(4) $2yz$

> **(1)** $\frac{1}{2yz}$ $\quad \frac{4xyz}{4yz} = \frac{2}{4yz} = \frac{1}{2yz}$

20. The equation $\mathbf{V = \frac{1}{3}BH}$ is equivalent to:

(1) $H = \frac{3V}{B}$

(3) $H = \frac{B}{3V}$

(2) $H = \frac{1}{3}BV$

(4) $H = 3V - B$

> **(1)** $H = \frac{3V}{B}$
>
> $3 \times V = \frac{1}{3}BH \times 3$
>
> $3V = BH$
>
> $\frac{3V}{B} = \frac{BH}{B}$
>
> $\frac{3V}{B} = H$

21. The formula $\mathbf{w = \frac{4e^3}{5}}$ is used to calculate the approximate weight, **w**, in grams of an ice cube with edges that are **e** centimeters long. Solve the formula for **e**.

> $\frac{w}{1} = \frac{4e^3}{5}$
>
> $\frac{5w}{4} = \frac{4e^3}{4}$
>
> $\frac{5w}{4} = e^3$ Take the cube root on both sides → $\sqrt[3]{\frac{5w}{4}} = e$

22. The distance a free falling object has traveled can be modeled by the equation $\mathbf{d = \frac{1}{2}at^2}$ where **a** is acceleration due to gravity and **t** is the amount of time the object has fallen.

a) Express **t** in terms of **a** and **d**.

2 x $d = \frac{1}{2}at^2$ **x 2** — Multiply by 2 on both sides to eliminate the fraction

$\frac{2d}{a} = \frac{at^2}{a}$ — Divide by **a** on both sides

$\frac{2d}{a} = t^2$

$\sqrt{\frac{2d}{a}} = \sqrt{t^2}$ — Take the square root on both sides

$\mathbf{\sqrt{\frac{2d}{a}} = t}$

b) Find the amount of time an object spent falling if it accelerated 2.5 inches/sec^2 and traveled a distance of 20 inches.

The amount of time can be found using the original equation of the <u>equivalent equation</u> as shown below. Substitute a and d $\rightarrow$ a = 2.5 and d = 20

$\sqrt{\frac{2d}{a}} = t$

$\sqrt{\frac{2(20)}{2.5}} = t$

$\sqrt{\frac{40}{2.5}} = t$

$\sqrt{16} = t$

$\mathbf{4 = t}$

$d = \frac{1}{2}at^2$

$20 = \frac{1}{2}(2.5)t^2$

$20 = 1.25t^2$

$\frac{20}{1.25} = \frac{1.25t^2}{1.25}$

$16 = t^2$

$\mathbf{4 = t}$

The object was falling for 4 seconds.

23. Solve the equation. For each step, list the property used.

3x – [8 – 3(x – 1)] = x + 19

3x – [8 – 3x + 3] = x + 19	**Distributive Property** (distributed – 3)
3x – (-3x + 11) = x + 19	Combined Like Terms (**Associative/Commutative Properties of +**)
3x + 3x – 11 = x + 19	**Distributive Property** (distributed the – sign)
6x – 11 = x + 19	Combined Like Terms (**Associative Property of +**)
5x – 11 = 19	**Subtraction Property of Equality** (subtracted x on both sides)
5x = 30	**Addition Property of Equality** (added 11 to both sides)
x = 6	**Division Property of Equality** (divided by 5 on both sides)

24. Find the value of the variable that makes each statement true.

a) $\frac{7a-5}{3}=\frac{9a}{4}$

3(9a) = 4(7a – 5)

27a = 28a – 20
-28a -28a

$\frac{-a}{-1} = \frac{-20}{-1}$

a = 20

Check

$\frac{7(20)-5}{3} = \frac{9(20)}{4}$

$\frac{135}{3} = \frac{180}{4}$

45 = 45

Always check your solution. Ask yourself, "does the value of the variable make the equation true?"

b) $\frac{1}{6}+\frac{x-2}{3}=\frac{5}{6}$

1st Method: Multiply by LCD

$\mathbf{6}\times\left(\frac{1}{6}+\frac{x-2}{3}\right)=\left(\frac{5}{6}\right)\times\mathbf{6}$

$\overset{1}{\cancel{6}}\left(\frac{1}{\cancel{6}}\right)+\overset{2}{\cancel{6}}\left(\frac{x-2}{\cancel{3}}\right)=\overset{1}{\cancel{6}}\left(\frac{5}{\cancel{6}}\right)$

1 + 2(x – 2) = 5

1 + 2x – 4 = 5
-3 + 2x = 8
2x = 8
x = 4

2nd Method: Create a Proportion

$\frac{1}{6}+\frac{x-2}{3}=\frac{5}{6}$

$-\mathbf{\frac{1}{6}} \qquad -\mathbf{\frac{1}{6}}$

$\frac{x-2}{3}=\frac{4}{6}$

6(x – 2) = (3)(4)
6x – 12 = 12
6x = 24
x = 4

25. Olivia wants to purchase some fruit. She wants to buy twice as many bananas as apples and three times as many oranges as apples. Each apple sells for \$0.32, a banana sells for \$0.15 and an orange sells for \$0.45. If Olivia has \$10 to spend, which equation can she use to find the number of apples, **x**, she can purchase?

(1) 0.32x + 0.15(2x) + 0.45(3x) = 10

(2) 0.32x + 0.45(2x) + 0.15(3x) = 10

(3) x + 2x + 3x = 10

(4) 0.32(3x) + 0.45x + 0.15(2x) = 10

x: the number of apples
2x: the number of bananas
3x: the number of oranges

Multiply the price by the quantity to find the total amount

Item	Quantity	Value (price per fruit)	Total Value (amount of \$ spent)
apples	x	\$0.32	.32x
bananas	2x	\$0.15	.15(2x)
oranges	3x	\$0.45	.45(3x)