PIPS – Linear Systems Answer Key

1. Examine the system below. Which algebraic method (*elimination* or *substitution*) would you use to solve the system? **Explain your reasoning** to your group members. Have half your group solve the system using one algebraic method and the other half of the group solve using the other algebraic method. Did you arrive at the same answer?

$-3x + 3y = 3$
$-5x + y = 13$

Solution
(-3, -2)

Elimination

$-3x + 3y = 3 \rightarrow -3x + 3y = 3$
$-3(-5x + y = 13) \rightarrow 15x - 3y = -39$

$12x + 0 = -36$

$\frac{12x}{12} = \frac{-36}{12}$

$x = -3$

$-5x + y = 13$
$-5(-3) + y = 13$
$15 + y = 13$
$y = -2$

Substitution

$-3x + 3y = 3 \rightarrow -3x + 3y = 3$
$-5x + y = 13 \rightarrow y = 5x + 13$

$-3x + 3(5x + 13) = 3$
$-3x + 15x + 39 = 3$
$12x + 39 = 3$
$\frac{12x}{12} = \frac{-36}{12}$
$x = -3$

$-5x + y = 13$
$-5(-3) + y = 13$
$15 + y = 13$
$y = -2$

2. Construct a system of two linear equations where (0, 5) is a solution to the first equation but is not a solution to the second equation and (3, 8) is the solution to the system. Graph the system in order to justify that the system you created satisfies the given conditions.

First Equation:
(0,5) (3,8)

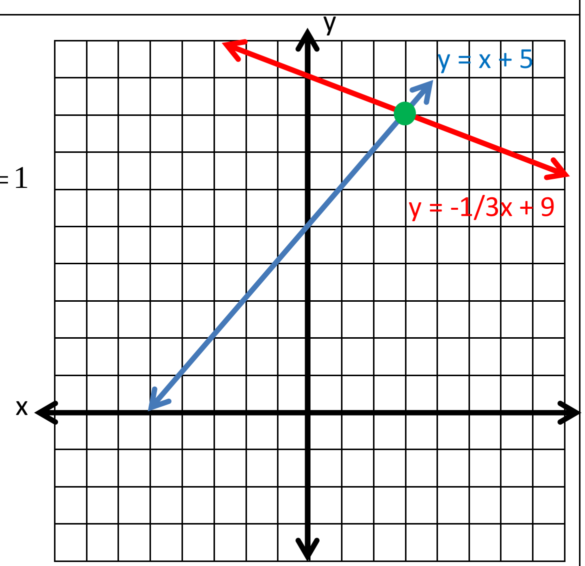
$$\frac{\Delta y}{\Delta x} = \frac{8-5}{3-0} = \frac{3}{3} = 1$$

m =1 b = 5

y = x + 5

Second Equation:
Any equation that runs through the point (3, 8) except y = x + 5

Example: **$y = -\frac{1}{3}x + 9$**

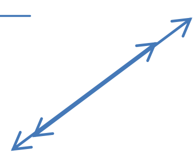


3. Solve each system below <u>algebraically</u>. Describe the solution set. Describe what each system looks like <u>graphically</u>.

a) $2y - 2x = 6$
$3y - 3x = 9$

$3(2y - 2x = 6) \rightarrow 6y - 6x = 18$
$-2(3y - 3x = 9) \rightarrow -6y + 6x = -18$

$0 + 0 = 0$



Infinite Solutions

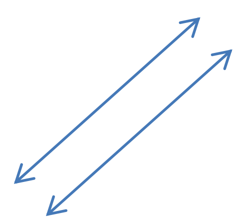
The lines coincide (lie on top of one another)

y = x + 3, y = x + 3

b) $2x = y - 4$
$3y = 6x$

$2x = y - 4 \rightarrow y = 2x + 4$

$3y = 6x$
$3(2x + 4) = 6x$
$6x + 12 = 6x$
$-6x \quad -6x$
$12 \neq 0$



No Solution

The lines are parallel *y = 2x + 4, y = 2x*

4. At a state fair, there is a game where you throw a ball at a pyramid of cans. If you knock over all the cans, you win a prize. The cost is 3 throws for $1, but if you have an armband, you get 6 throws for $1. The armband is purchased at the entrance for $10.

a) Complete the table below that compares the cost of playing the game with an armband and without an armband.

x **# of throws**	y **Cost without armband**	y **Cost with armband**
0	0	10
6	2	11
12	4	12
18	6	13
24	8	14
30	10	15
36	12	16
42	14	17
48	16	18
54	18	19
60	**20**	**20**
66	22	21
72	26	22
78	28	23

b) Does it make sense to buy the armband if you want to play this game? **Explain** your reasoning.

If you plan on throwing the ball over 60 times, then you should get the armband because it's cheaper. However, if you are going to throw less than 60 times, it is cheaper to play without the armband. If you are going throw the ball 60 times, it wouldn't matter whether you had the armband or not since the cost is the same for both.

c) Write a system of equations that models the situation.

Without Armband: (0, 0) (6, 2)

$$\frac{\Delta y}{\Delta x} = \frac{2-0}{6-0} = \frac{2}{6} = \frac{1}{3} = \frac{\$1}{3 \text{ throws}}$$ **M** = 1/3 **B** = 0

With Armband: (0, 10) (6, 11)

$$\frac{\Delta y}{\Delta x} = \frac{11-10}{6-0} = \frac{1}{6} = \frac{\$1}{6 \text{ throws}}$$ **M** = 1/6 **B** = 10

Without Armband: $\mathbf{y = \frac{1}{3}x}$

With Armband: $\mathbf{y = \frac{1}{6}x + 10}$

What is the meaning of the rate of change (slope) and y-intercept in each equation?

$\mathbf{y = \frac{1}{3}x}$ **ROC**: 3 throws for a $1 or *about* 33¢ for one throw

Y-int: no additional cost ($0)

$\mathbf{y = \frac{1}{6}x + 10}$ **ROC**: 6 throws for $1 or *about* 17¢ for one throw

Y-int: pay $10 for an armband

d) What is the solution to the system? What does it mean in the context of the situation? **(60, 20)**

With 60 throws, a person with an armband and a person without an armband will spend the same amount ($20).