Unit 1 Review: The Real Number System

1. Using the words **natural**, **whole**, **integer**, **rational** and **irrational**, list all the subsets of real numbers to which each number belongs.

   a) 115 natural, whole, integer, rational

   b) $29.\overline{2}$ rational

   c) $\sqrt{10}$ irrational $\sqrt{10} \approx 3.16227...$

   d) $\sqrt[3]{-27}$ integer, rational $\sqrt[3]{-27} = -3$

2. Write two numbers that fit each description. If there is no such number, write *none*.

   a) negative integer -9, -35

   b) negative rational number that is not an integer -15.75, -3 ½

   c) irrational integer NONE

   d) negative irrational number $-\sqrt{15}$, $-\pi$

True/False. If false, explain why the statement is false.

3. $-\sqrt{25}$ is a rational number. **True** $-\sqrt{25} = -5$

   The number is equivalent to -5 and -5 is an integer, making it a rational number.

4. $\sqrt[3]{9}$ is a rational number. **False** $\sqrt[3]{9} \approx 2.08008382...$

   This number is irrational because it is a non-terminating, non-repeating decimal.

   It is also irrational because the number 9 is not a perfect cube number.

5. The sum of two irrational numbers is always an irrational number. **False**

   The sum can be rational or irrational. $\sqrt{2} + \sqrt{2} = 2\sqrt{2} \leftarrow irrational \ \ sum$

   $-\sqrt{2} + \sqrt{2} = 0 \leftarrow rational \ \ sum$

6. The product of two irrational numbers is always an irrational number. **False**

   The product can be rational or irrational. $\sqrt{2} \bullet \sqrt{5} = \sqrt{10} \leftarrow irrational \ \ product$

   $\sqrt{2} \bullet \sqrt{2} = \sqrt{4} \leftarrow rational \ \ product$

7. State whether the numbers below are **rational** or **irrational**.

a) $\sqrt{144}$
Rational

$\sqrt{144} = 12$

b) $\sqrt[3]{-64}$
Rational

$\sqrt[3]{-64} = -4$

c) $\sqrt{30}$
Irrational

$\sqrt{30} \approx 5.4772...$

d) $\sqrt[3]{18}$
Irrational

$\sqrt[3]{18} \approx 2.6207...$

8. Rewrite each square root radical in simplest radical form.

a) $\sqrt{24}$

$\sqrt{24}$
$\sqrt{4} \cdot \sqrt{6}$
$2\sqrt{6}$

$\mathbf{\sqrt{24} = 2\sqrt{6}}$

b) $\sqrt{32}$

$\sqrt{32}$
$\sqrt{16} \cdot \sqrt{2}$
$4\sqrt{2}$

$\mathbf{\sqrt{32} = 4\sqrt{2}}$

c) $\sqrt{63}$

$\sqrt{63}$
$\sqrt{9} \cdot \sqrt{7}$
$3\sqrt{7}$

$\mathbf{\sqrt{63} = 3\sqrt{7}}$

d) $\sqrt{108}$

$\sqrt{108}$
$\sqrt{36} \cdot \sqrt{3}$
$6\sqrt{3}$

$\mathbf{\sqrt{108} = 6\sqrt{3}}$

Calculator Check:
In order to make sure the original expression and the simplified expression are equivalent, type each expression into your calculator and press Enter. If the same non-terminating, non-repeating decimal appears, then the expressions are equivalent.

Identify the property shown by the statement below.

9. 14 + -15 = -15 + 14
Commutative of Add

10. $xy + xz = x(y + z)$
Distributive

11. $a + 0 = a$
Identity of Add

12. $b \bullet \frac{1}{b} = 1$
Inverse of Mult

13. -2 + (5 + 1) = (-2 + 5) + 1
Associative of Add

14. (-3)(4) = (4)(-3)
Commutative of Mult

15. $x + -x = 0$
Inverse of Add

16. 3.5 • (2 • 1.2) = (3.5 • 2) • 1.2
Associative of Mult

17. 8 • 1 = 8
Identity of Mult

Multiple Choice

18. If **S** is a non-zero rational number and **T** is an irrational number, which expression(s) will always result in an irrational number?

I. **S + T**

II. **ST**

III. $\mathbf{T^2}$

T^2 may result in a **rational** or **irrational** product.

If T = π then $T^2 = \pi^2$. Pi squared is irrational.

If T = $\sqrt{5}$ then $T^2 = \sqrt{25}$ and $\sqrt{25} = 5$. Five is an integer, therefore, it is a rational number.

(1) I (2) II **(3)** I and II only (4) I, II and III

19. Which set of numbers best describes the different amounts of ingredients that are possible to use in a recipe?

(1) whole numbers **(2)** positive rational numbers

(3) integers (4) real numbers

(1) partial quantities are possible, i.e. 1½ cups of flour
(3) & (4) negative quantities are not possible

Extended Response

20. Ernie says that it's possible for the product of two irrational numbers to be rational. State whether or not you agree with Ernie. Provide mathematical evidence to support your statement.

I agree with Ernie. It is possible for the product of two irrational numbers to be rational. See examples below.

$\sqrt{5} \bullet \frac{1}{\sqrt{5}} = 1$ Any number times its reciprocal always equals 1.

$\sqrt{2} \bullet \sqrt{32} = \sqrt{64}$ The result $\sqrt{64}$ is equivalent to 8 which is a rational number.

21. The process of showing that 2x + 3x is equivalent to 5x is shown below. Identify the property used in each step.

Step 1: 2x + 3x Given expression

Step 2: x (2 + 3) Distributive

Step 3: x(5) Combine terms

Step 4: 5x Commutative Property of Multiplication

The steps above prove that the expression 2x + 3x is the same as 5x. Prove the equivalence of these two expressions in another way.

Another way to prove that 2x + 3x is equivalent to 5x is by substituting the variable x with a number.

Let x = **10**

2x + 3x = 5x
2(**10**) + 3(**10**) = 5(**10**)
20 + 30 = 50
50 = 50

To test for equivalence, substitute any number in place of x. However, never use 1 or 0. The number 1 is the identity element of multiplication and any number multiplied by 0 = 0. If your result is the same for each expression then those expressions are equivalent.

22. The following flow diagram shows that **a(bc)** is equivalent to **b(ac)**.