8 Algebra CC
Unit 12 Review – Exponential Functions ANSWER KEY

Important Terminology

exponential function
interval
growth rate
initial value
exponential growth
decay rate
common ratio (growth/decay factor)
exponential decay

Exponential Function: $y = ab^x$ **Growth Model**: $y = a(1 + r)^t$ **Decay Model:** $y = a(1 – r)^t$

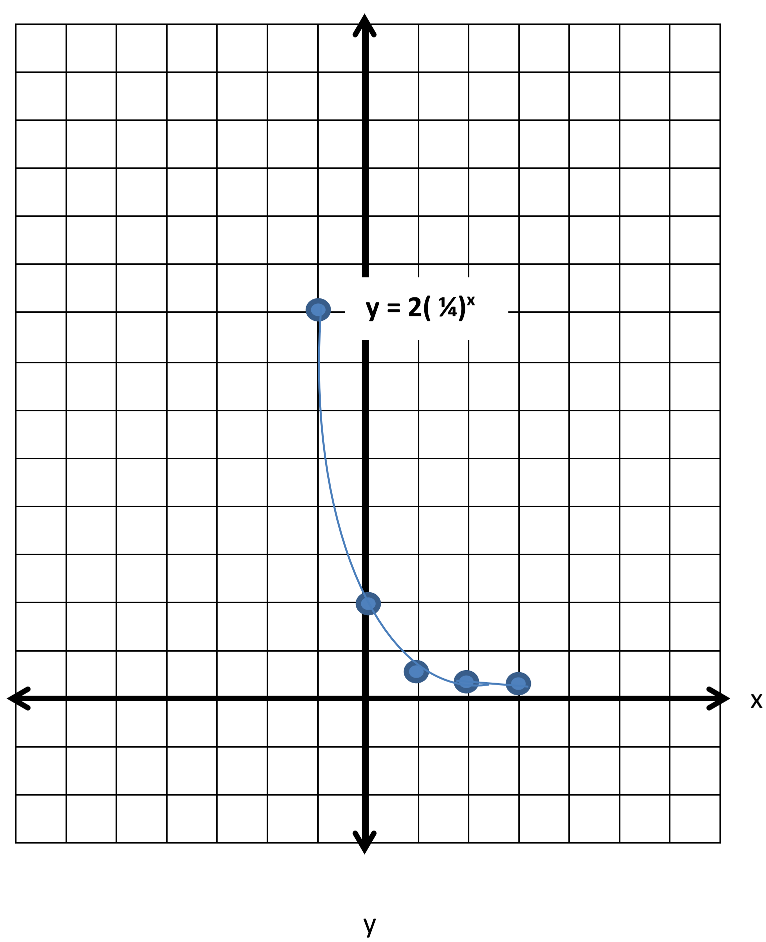
What should I be able to do?

1. Graph exponential functions with restricted and unrestricted domains.
2. Determine if an exponential function is increasing or decreasing from a table, graph and equation.
3. Identify the y-intercept and common ratio of an exponential function from a table, graph and equation.
4. Determine the average rate of change of a function over a specified interval.
5. Model situations/relationships with exponential functions.
6. Solve problems using an exponential growth or decay model.
7. Determine if a function is linear or exponential by examining a table, graph or equation.
8. Determine if a situation or set of data is best modeled by a linear or exponential function.

Practice Problem Set

1. Graph the exponential function $y = 2(¼)^x$ given the *domain* $-1 \le x \le 3$.

x	y
-1	8
0	2
1	½ = 0.5
2	1/8 = 0.125
3	1/32 = 0.03125



Remember to label everything!
No arrows!!

2. Identify the **y-intercept** of each function below and state whether the function will ***increase*** or ***decrease*** when graphed.

a) $f(x) = 34(2.75)^x$

y-int: 34
Increase

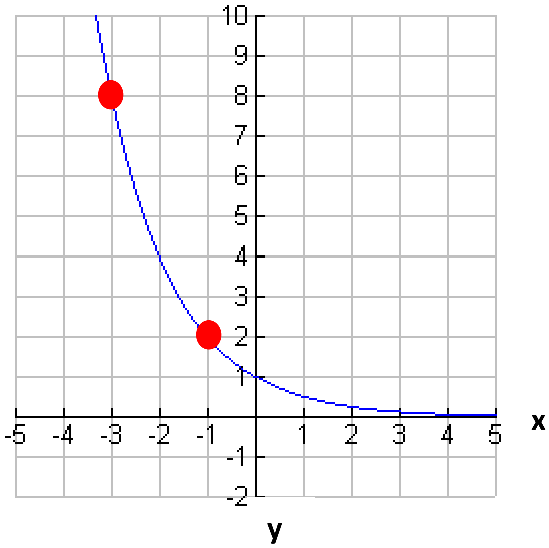
b) $f(x) = (0.25)^x$

y-int: 1
Decrease

c) $f(x) = \frac{2}{3}\left(\frac{3}{2}\right)^x$

y-int: 2/3
Increase

3. Which function is decreasing at a faster rate over the interval $-3 \le x \le -1$?



x	y
-5	21.5
-4	18
-3	**14.5**
-2	11
-1	**7.5**

Graph
Begin: (-3, 8)
End: (-1, 2)

Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{8-2}{-3-(-1)} = \frac{6}{-2} = -3$$

Table
Begin: (-3, 14.5)
End: (-1, 7.5)

Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{14.5-7.5}{-3-(-1)} = \frac{7}{-2} = -3.5$$

Over the specified interval, the function displayed in the table is decreasing at a faster rate. This is because the rate of change is 3.5 units per 1 unit. The graph displays a rate of change of 3 units per 1 unit and 3.5 > 3.

4. A pharmaceutical company has tested a new time-release cold pill. It finds that the amount of milligrams, ***f(n)***, of the active ingredients of the pill left in the bloodstream ***n*** hours after it is taken can be estimated using the function $f(n) = 35(0.87)^n$.

a) How many milligrams of cold medicine are in the pill before it is swallowed?

35 milligrams

$y = a(1 - r)^t$
a: initial amount
r: rate of decrease
1 – r: common ratio (decay factor)
t: time

b) What percent of the drug leaves the body each hour?

13%

c) How many milligrams, *to the nearest thousandth*, of the cold medicine remain in the body after 5 hours have passed?

$f(n) = 35(0.87)^n$
$f(5) = 35(0.87)^5$
$= 17.4447\ldots$
≈ 17.445 **milligrams**

5. Find the balance of an account after 5 years that pays 5.2% interest compounded yearly with an initial investment of $1250.

$y = a(1 + r)^t$
$y = 1250(1 + 0.052)^t$
$y = 1250(1.052)^5$
$y = 1610.603....$ $1610.60

6. Between 1990 and 2000, the profits of a business decreased approximately 0.7% each year. In 1990, the business's profit was $1.4 million. What was the profit in 1996?

$y = a(1 - r)^t$
$y = 1{,}400{,}000(1 - 0.007)^t$
$y = 1{,}400{,}000(0.993)^6$ ←*1990 to 1996 is 6 years*
$y = 1{,}342{,}219.446...$ $1,342,219.45

7. A construction company purchased a piece of equipment valued at $300,000. The value of the equipment depreciates at a rate of 14% per year.

a) Write an equation that determines the approximate value of the equipment each year.

$y = a(1 - r)^t$
$y = 300{,}000(1 - 0.14)^t$
$y = 300{,}000(0.86)^t$

b) What is the value of the equipment after 9 years?

$y = 300{,}000(0.86)^t$
$y = 300{,}000(0.86)^9$
$y = 77{,}198.225...$ $77,198.23

c) Estimate when the equipment will have a value of $50,000.

x	y
11	57096
12	49102

Pictured to the left is year 11 and year 12 (*I accessed this information using the table from my calculator*).

The equipment will be valued at $50,000 in between the two years, closer to 12.

In about 12 years, the equipment will be worth $50,000.

8. A business publication recently reported profit trends of four computer companies over the past decade. Each company's earnings can be summarized by the functions below labeled A – D where ***P(t)*** represents the profit over ***t*** years.

A. $P(t) = 1{,}430{,}000(1.29)^t$ **B.** $P(t) = 2{,}345{,}000(0.76)^t$

C. $P(t) = 1{,}987{,}000(1.095)^t$ **D.** $P(t) = 2{,}000{,}500(0.695)^t$

I. Which companies are making money? **Companies A and C**

II. Which companies are losing money? **Companies B and D**

III. What are the ***growth rates*** of each company that is making money?

Company A: 0.29 (29%) – Profits increase each year by 29% each year.

Company C: 0.095 (9.5%) – Profits increase each year by 9.5%.

IV. What are the ***decay rates*** of each company that is losing money?

Company B: 0.24 (24%) – Profits decrease each year by 24% each year.

Company D: 0.305 (30.5%) – Profits decrease each year by 30.5%.

(*In order to get the decay rate, subtract the common ratio from 1*)

1 – 0.76 = 0.24
1 – 0.695 = 0.305

V. Which company is earning money the fastest? Which company is losing money the fastest?

Company A is earning money the fastest because the growth rate is greater than that of Company C. 29% > 9.5%

Company D is losing money faster than company B because the decay rate is greater. 30.5% > 24%

9. What type of function best models the situation?

A. Gregory plans to purchase a video game player. He has $500 in his savings account and plans to save $20 per week from his allowance until he has enough money to buy the player.

Linear
Reason: There is a constant rate of change (+ 20 each week)

B. City workers recorded the number of squirrels in a park over a period of time. At the first count, there were 15 pairs of male and female squirrels (*30 squirrels total*). After 6 months, the city workers recorded a total of 60 squirrels and after a year there were 120 squirrels.

Exponential
Reason: The population grows by a common ratio (x 2)

10. According to the International Basketball Federation (FIBA), a basketball must be inflated to a pressure such that, when it is dropped from a height of 1,600 mm, it will rebound to a height of 1,200 mm. Maddie decides to test the rebound ability of her new basketball. She assumes that the ratio of each rebound height to the previous rebound height remains the same. Let ***f(n)*** be the height of the basketball after ***n*** bounces.

a) Complete the chart below to reflect the heights Maddie expects to measure.

n	*f(n)*
0	1600
1	1200
2	**900**
3	**675**
4	**506.25**

The common ratio is $\frac{1200}{1600} = 0.75$

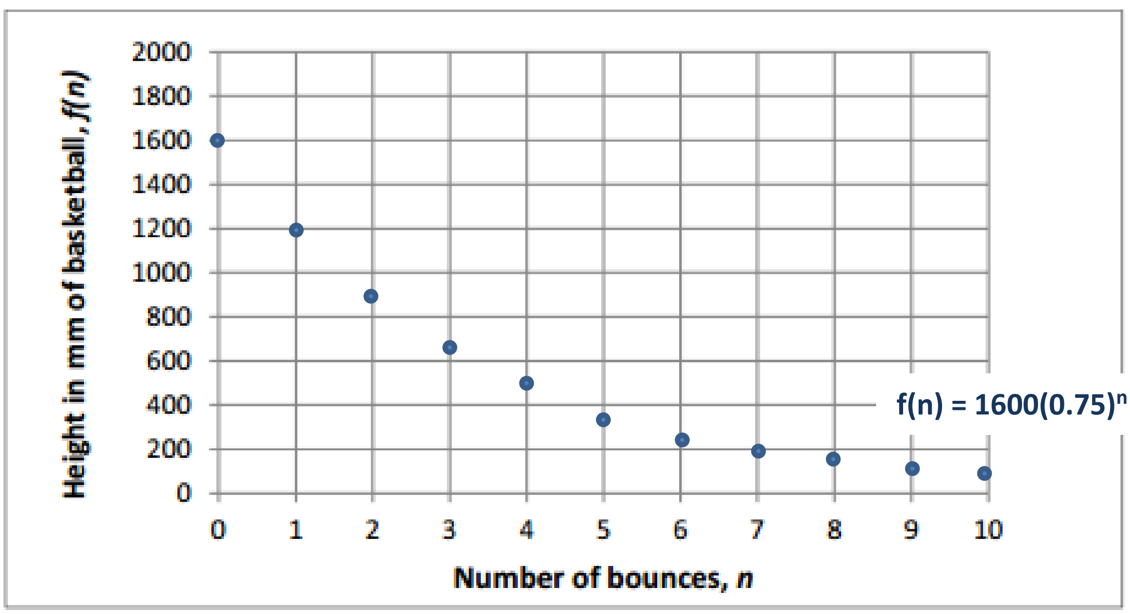
Multiply 1600 by 0.75 and so on.

This means the height of the ball reaches 75% of its previous height after each bounce.

b) Write an exponential function that models this situation.

$f(n) = 1600(0.75)^n$ *or* $f(n) = 1200(0.75)^{n-1}$

c) Graph the function on the grid below from 0 to 10 bounces. Using the curve created, estimate the bounce number at which the rebound height will drop below 200 mm.



n (bounces)	f(n) (height)
0	1600
1	1200
2	900
3	675
4	506
5	380
6	285
7	214
8	160
9	120
10	90

On the 8th bounce, the ball will rebound to a height below 200 mm. This can be seen on the graph and from the table pictured to the right.
(Note: Do not connect the points since there are only whole number bounces possible!)