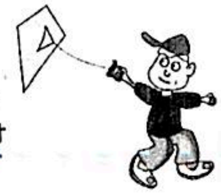
**Essential Question:** How do we calculate and interpret the rate of change?

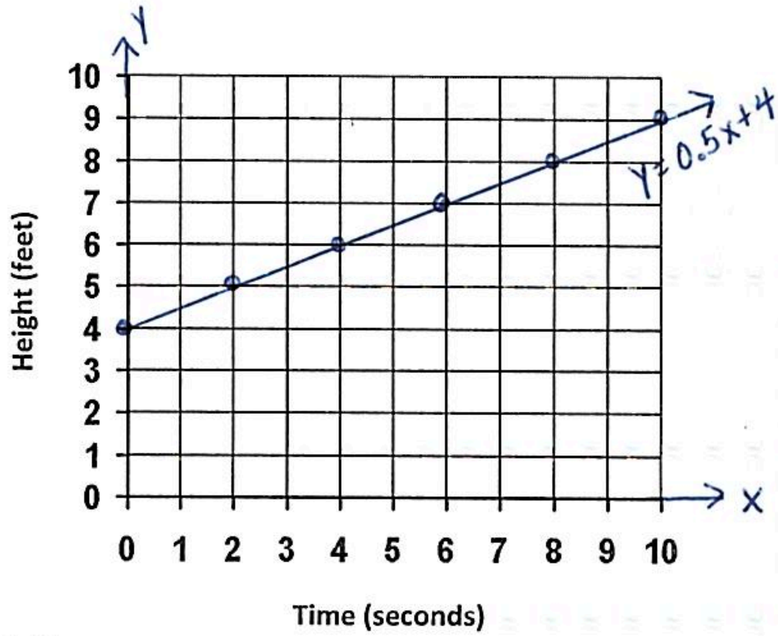
**Do Now:**



A person is flying a kite in a park. The height of the kite as the string is let out can be modeled by the equation $\boxed{y = 0.5x + 4}$ where y represents the height of the kite in feet and x represents the number of seconds that passes as the string is let out.

a) Complete the table of values.

| Seconds ($x$) | Height ($y$) |
|---|---|
| 0 | 4 |
| 2 | 5 |
| 4 | 6 |
| 6 | 7 |
| 8 | 8 |
| 10 | 9 |




b) Graph the relationship between height and time.

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**Let's take a closer look at the linear relationship from the Do Now.**

**What is the y-intercept? What does it mean in the context of the situation?**

y int: 4    sec ft (0, 4)    Initially, the kite was at a height of 4 feet.

**What is the slope of the line? What does it mean in the context of the situation?**

$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{1 \text{ ft}}{2 \text{ sec}}$ The kite rises at a rate of 1 foot every 2 seconds

or $\frac{1}{2} = \frac{.5 \text{ ft}}{1 \text{ sec}}$ The kite goes up at a rate of 1/2 foot per second.

## Calculating and Interpreting a Rate of Change

The **rate of change** is a ratio that compares the amount of change in a dependent variable ($\Delta y$) to the amount of change in an independent variable ($\Delta x$). Linear Relationships show a **constant** rate of change.

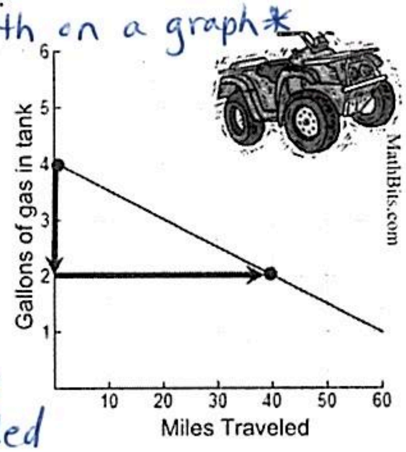
$$\frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}}$$

vertical change ← dependent

horizontal change ← independent

1) Courtney's ATV had 4 gallons of gasoline in the tank on Monday. After driving a total of 40 miles during the week, she had 2 gallons of gas remaining.

* Check how much each box is worth on a graph *




a) Calculate the ***rate of change*** of the line.

$$\frac{\Delta y}{\Delta x} = \frac{-2}{40} = \frac{-1}{20}$$

$$= \frac{-.05}{1}$$

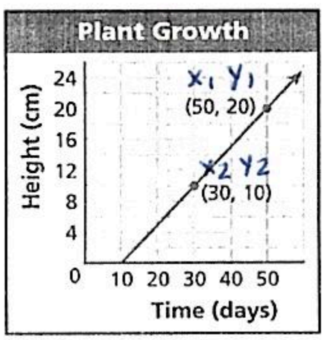
b) What does the rate of change tell us?

$$\frac{\Delta y}{\Delta x} = \frac{-1}{20}$$ The ATV uses 1 gallon of gas for every 20 miles traveled

$$= \frac{-.05}{1}$$ The ATV uses .05 gallon of gas per mile.

2) Calculate the rate of change of each line. What does the ratio tell us?

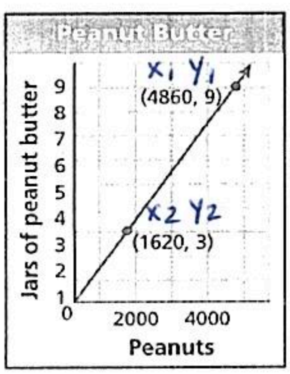
a)




$$\frac{\Delta y}{\Delta x} = \frac{20-10}{50-30} = \frac{10}{20} = \frac{.5 \text{ cm}}{1 \text{ days}}$$

The plant grows at a rate of .5 cm per day.

b)

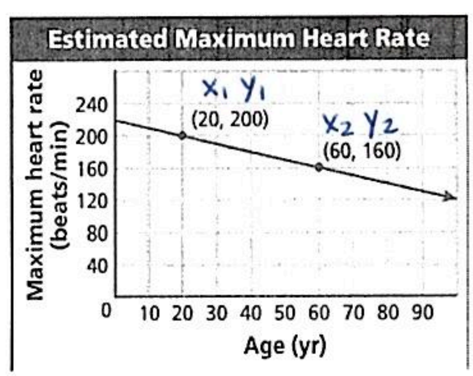



$$\frac{\Delta y}{\Delta x} = \frac{9-3}{4860-1620} = \frac{6}{3240} = \frac{.00\overline{185} \text{ jars}}{1 \text{ peanuts}}$$

One peanut fills $.00\overline{185}$ of a jar

$$\frac{6}{3240} = \frac{1 \text{ jar}}{540 \text{ peanuts}}$$ One jar holds 540 peanuts

3) The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Determine the rate of change and explain what the rate of change means in the context of the situation.



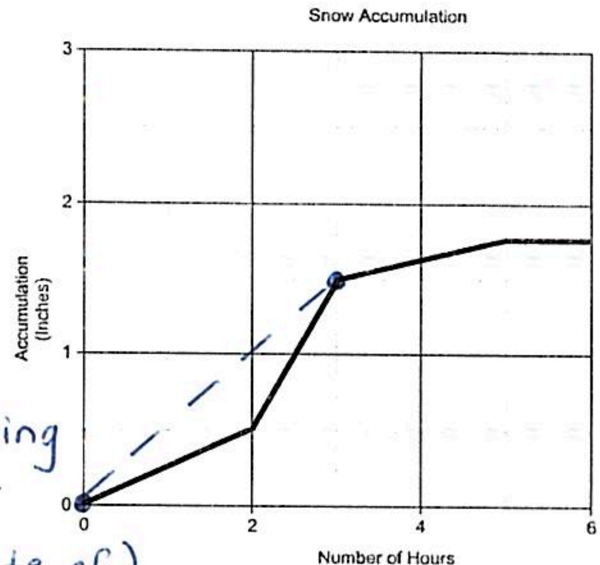

$$\frac{\Delta y}{\Delta x} = \frac{200-160}{20-60} = \frac{40}{-40} = \frac{1}{-1} = \frac{-1 \text{ heartbeat per min}}{1 \text{ age (years)}}$$

Your heartbeat decreases by one heartbeat per minute every year you get older.

## Think about this... *Is the rate of change always constant?*

Not every situation has a constant rate of change. Sometimes we need to calculate the ***average*** rate of change.

Consider the graph below which shows snow accumulation over a six hour period of time. What is the average rate of change of snow accumulation from 0 – 3 hours?




$(0,0) \quad (3,1.5)$

$$\frac{\Delta y}{\Delta x} = \frac{0-1.5}{0-3} = \frac{-1.5}{-3} = 0.5$$

$$\frac{\Delta y}{\Delta x} = \frac{0.5 \text{ inches}}{1 \text{ hours}} =$$

snow is accumulating $\frac{1}{2}$ inch per hour

(at an average rate of change from 0 to 3 seconds)

4) Within 10 seconds of a dive, a scuba instructor finds himself 30 feet below the surface of the water. After 40 seconds, he reaches a depth 100 feet below the surface of the water. Determine the average rate at which the diver descends.

$\begin{matrix} x_1 & y_1 \\ (10, & -30) \end{matrix} \quad \begin{matrix} x_2 & y_2 \\ (40, & -100) \end{matrix}$

$$\frac{\Delta y}{\Delta x} = \frac{-100-(-30)}{40-(10)}$$

$$= \frac{-70}{30} = -\frac{7 \text{ feet below water surface}}{3 \text{ seconds}}$$

$$-\frac{7}{3} = \frac{-2\frac{1}{3} \text{ feet}}{1 \text{ seconds}}$$

The average rate of descent is $2\frac{1}{3}$ feet per second.

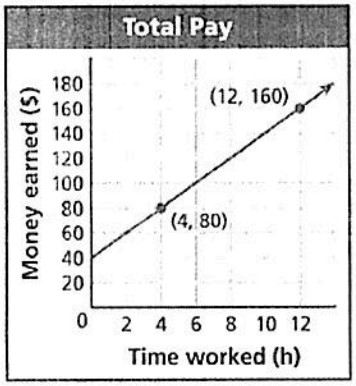
the instructor descends at an average rate of 7 feet below the water surface every 3 seconds.

## The TAKE AWAY

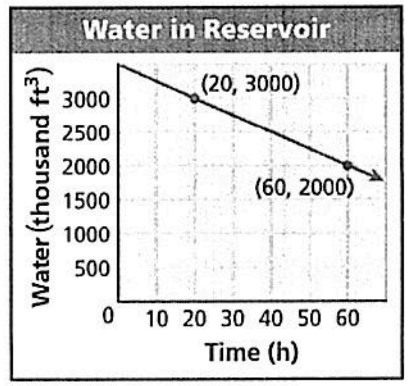
Use $\frac{\Delta y}{\Delta x}$ to find the rate of change. The graph that displays a constant rate of change is always a picture of a straight line (linear function). If the graph is not a straight line, we can calculate the average rate of change over a specific interval.

For each graph below, calculate the rate of change and explain its meaning.

1)




2)




3) Liam, the terrible toddler, was playing with the bathtub faucet when no one was looking. After two minutes, he had filled the tub with 12 gallons of water and after 4 minutes, the tub was filled with 20 gallons of water. Calculate the average rate at which water was entering the bathtub from 2 to 4 minutes.