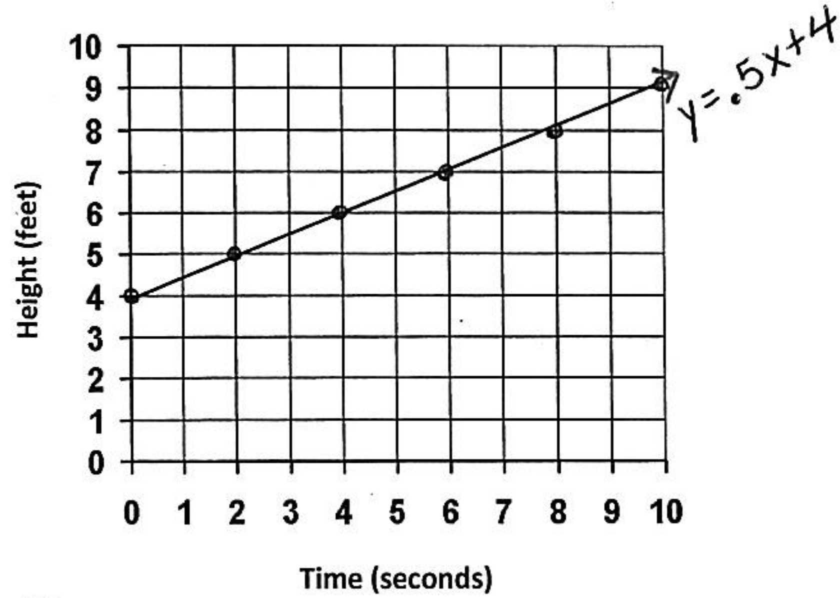
Essential Question: How do we calculate and interpret the rate of change?

Do Now:

A person is flying a kite in a park. The height of the kite as the string is let out can be modeled by the equation **y = 0.5x + 4** where **y** represents the height of the kite in feet and **x** represents the number of seconds that passes as the string is let out.

a) Complete the table of values.

Seconds (x)	Height (y)
0	4
2	5
4	6
6	7
8	8
10	9



b) Graph the relationship between height and time.

Let's take a closer look at the linear relationship from the Do Now.

What is the y-intercept? What does it mean in the context of the situation?

(0,4) Before flying the kite (0 seconds) the kite is at a height of 4 feet

What is the slope of the line? What does it mean in the context of the situation?

$\frac{\Delta y}{\Delta x}$ $\frac{1 \text{ foot}}{2 \text{ seconds}}$ the kite string is let out one foot every two seconds

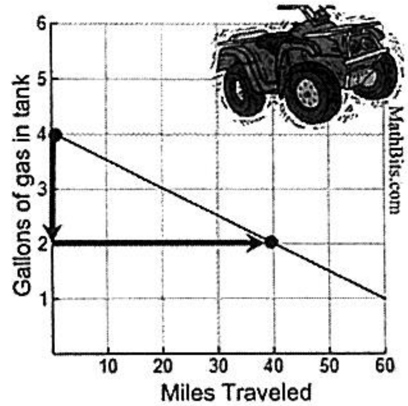
unit rate
make the denominator = 1
$\frac{1}{2} \rightarrow \frac{.5}{1}$ one-half foot of string is let out every second

Calculating and Interpreting a Rate of Change

The **rate of change** is a ratio that compares the amount of change in a dependent variable (Δy) to the amount of change in an independent variable (Δx). Linear Relationships show a **constant** rate of change.

$$\frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}}$$

1) Courtney's ATV had 4 gallons of gasoline in the tank on Monday. After driving a total of 40 miles during the week, she had 2 gallons of gas remaining.



a) Calculate the ***rate of change*** of the line.

$\frac{\Delta y}{\Delta x}$ $\frac{\text{gallons}}{\text{miles}}$ $\frac{-2}{40} \rightarrow \frac{.05}{1}$

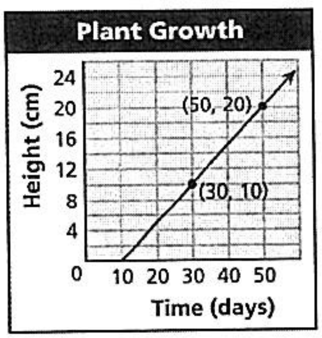
* be careful to check how much each box is worth *

b) What does the rate of change tell us?

$-\frac{.05}{1}$ $\frac{\text{gallon}}{\text{mile}}$ .05 gallons are used every mile

2) Calculate the rate of change of each line. What does the ratio tell us?

a)

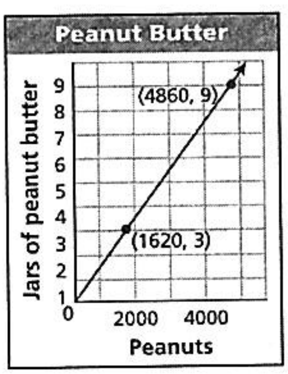


$\begin{matrix} x_1 & y_1 \\ (30, & 10) \end{matrix}$ $\begin{matrix} x_2 & y_2 \\ (50, & 20) \end{matrix}$

$\frac{\Delta y}{\Delta x}$ $\frac{\text{height (cm)}}{\text{time (days)}}$ $\frac{10-20}{30-50} \rightarrow \frac{-10}{-20} \rightarrow \frac{.5}{1}$

every day, a plant grows .5 cm

b)

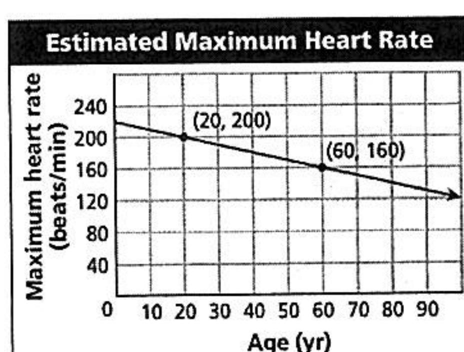


$\frac{\Delta y}{\Delta x}$ $\frac{\#\text{jars}}{\#\text{peanuts}}$ $\frac{3-9}{1620-4860} \rightarrow \frac{-6}{-3240} \rightarrow \frac{.001851}{1}$

one peanut fills approx .00185 jars

or $\frac{1}{540}$ one jar has 540 peanuts

3) The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Determine the rate of change and explain what the rate of change means in the context of the situation.



$\frac{\Delta y}{\Delta x}$ $\frac{\#\text{heartbeats a minute}}{\text{age in years}}$ $\frac{200-160}{20-60} \rightarrow \frac{40}{-40} \rightarrow \frac{1}{-1} \rightarrow \frac{-1}{1}$

$\frac{-1}{1}$ $\frac{\text{heartbeats per minute}}{\text{age in years}}$

every year you get older, your heartbeat per minute decreases by one

Think about this... ***Is the rate of change always constant?***

Not every situation has a constant rate of change. Sometimes we need to calculate the **average** rate of change.

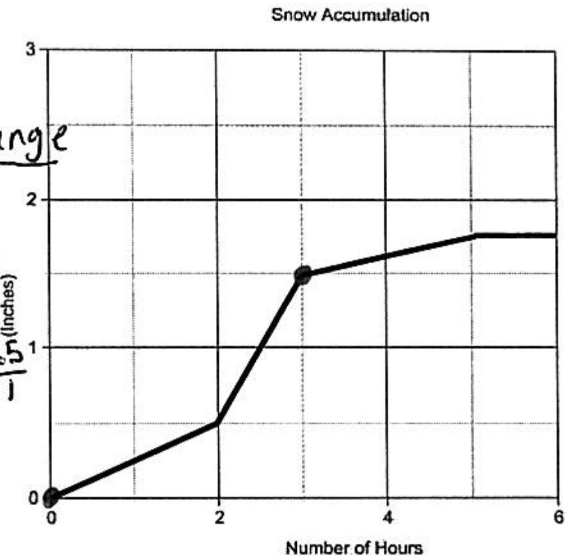
Consider the graph below which shows snow accumulation over a six hour period of time. What is the *average rate of change* of snow accumulation from 0 – 3 hours?

Find your starting point (0,0)
and your ending point (3, 1.5)

calculate the average rate of change
using only those two points

$\frac{\Delta y}{\Delta x}$ $\frac{\text{inches of snow}}{\text{number of hours}}$ $\frac{1.5-0}{3-0} \rightarrow \frac{1}{2} \rightarrow \frac{.5}{1}$

$\frac{1}{2}$ inch of snow falls per hour



4) Within 10 seconds of a dive, a scuba instructor finds himself 30 feet below the surface of the water. After 40 seconds, he reaches a depth 100 feet below the surface of the water. Determine the average rate at which the diver descends.

seconds ↓ ↙ feet below surface
starting point (10, -30)
ending point (40, -100)

$\frac{\Delta y}{\Delta x}$ $\frac{-100+30}{40-10} \rightarrow \frac{-70}{30} \rightarrow \frac{-2\frac{1}{3}}{1}$ the diver descends $2\frac{1}{3}$ feet every second

The TAKE AWAY

Use $\frac{\Delta y}{\Delta x}$ to find the rate of change. The graph that displays a constant rate of change is always a picture of a linear equation. If the graph is not a straight line, we can calculate the average rate of change over a specific interval.