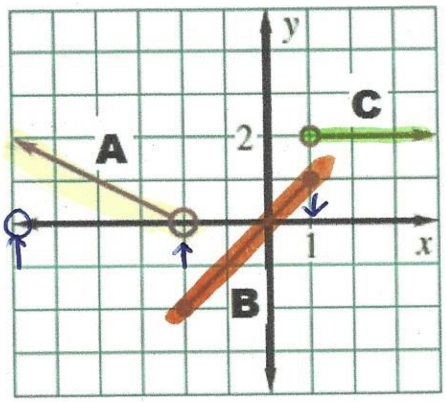
ALEGRBA RH

Essential Question: What are piecewise functions? How are they defined, graphed and applied to real life situations?

Do Now: State the domain for each line graphed below

x values



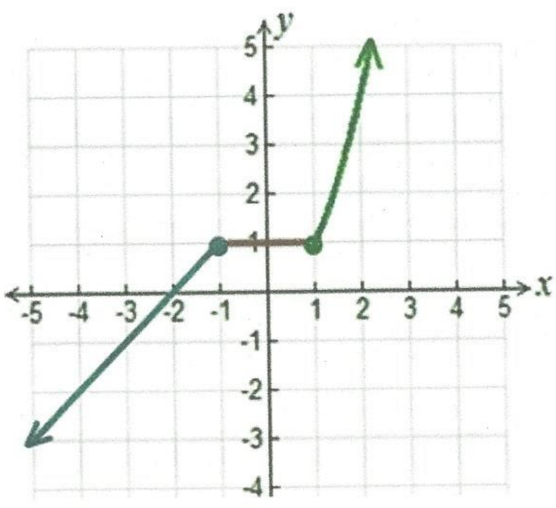
	inequality notation	interval notation
A.	$x < -2$	$(-\infty, -2)$
B.	$-2 \leq x \leq 1$	$[-2, 1]$
C.	$x > 1$	$(1, \infty)$

PIECEWISE FUNCTIONS

We have seen many graphs that are expressed as single equations and are continuous over a domain of real numbers. There are also graphs that are defined by "different equations". These graphs may be continuous, or they may contain "breaks". Because these graphs tend to look like "pieces" glued together to form a graph, they are referred to as **piecewise functions.**

A piecewise defined function is a function defined by at least two equations ("pieces"), each of which applies to a different part of the domain. Piecewise defined functions can take on a variety of forms. Their "pieces" may be all linear, or a combination of functional forms (such as constant, linear, quadratic, cubic, square root, cube root, exponential, etc.).

Example:



equation restricted domain

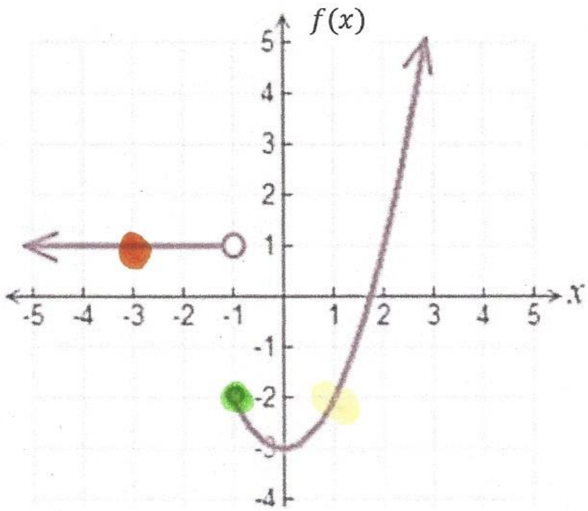
$$f(x)=\begin{cases} x+2; & x\leq -1 \\ 1; & -1<x<1 \\ x^2; & x\geq 1 \end{cases}$$

The piecewise function shown in this example is ***continuous*** (there are no "gaps" or "breaks" in the graph).

In this example, the domain is all real numbers since all x-values have a value.

Practice Problem Set

1. Given the following piecewise function $f(x)$:



a. State the domain:
x values

$(-\infty, \infty)$

$x \in \mathbb{R}$
(all real numbers)

b. State the range:
y values

$[-3, \infty)$

$y \geq -3$

c. Find $f(-3)$

1

d. Find $f(1)$

-2

e. Find $f(-1)$

-2

f. Is this function continuous or non-continuous?

there is a break (gap) between the pieces.

Defining Piecewise Functions

- Write an expression for each piece graphed over its domain.
- Write a definition for the graph, which is done by identifying the different domains shown in the graph.
- Remember: ○ for $<$ and $>$

  • for $\leq$ and $\geq$

g. Define the function graphed above

$$f(x) = \begin{cases} 1; & x < -1 \\ x^2 - 3; & x \geq -1 \end{cases}$$

↑ expression for each function

↑ domain → inequality notation always !!

2. Graph the following piecewise function and state the domain/range.

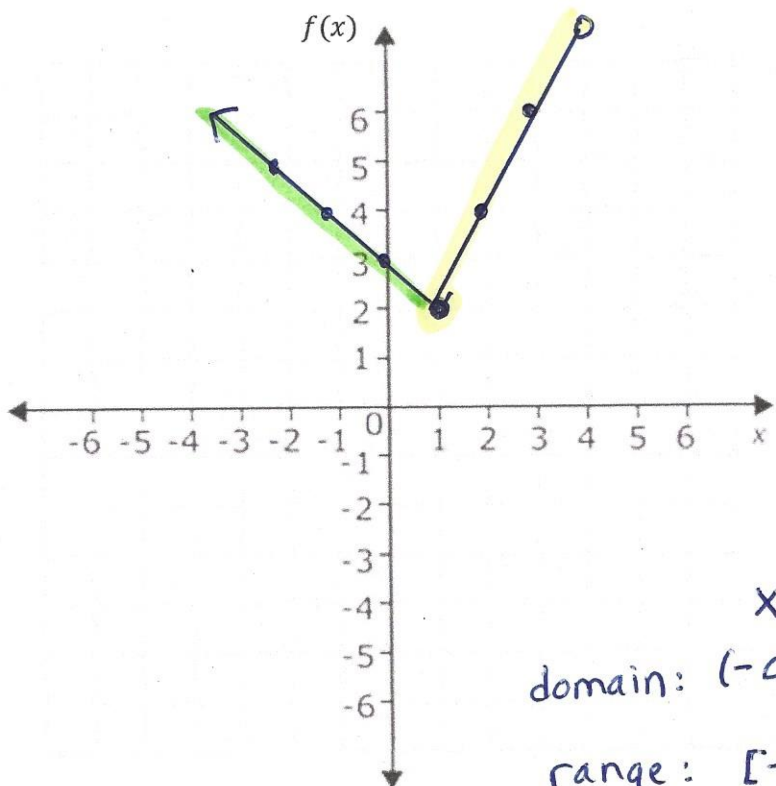
$$f(x) = \begin{cases} -x+3 & x<1 \\ 2x & 1 \le x < 4 \end{cases}$$

$y = -x+3$

x	y
1	2
0	3
-1	4
-2	5

$y = 2x$

x	y
1	2
2	4
3	6
4	8



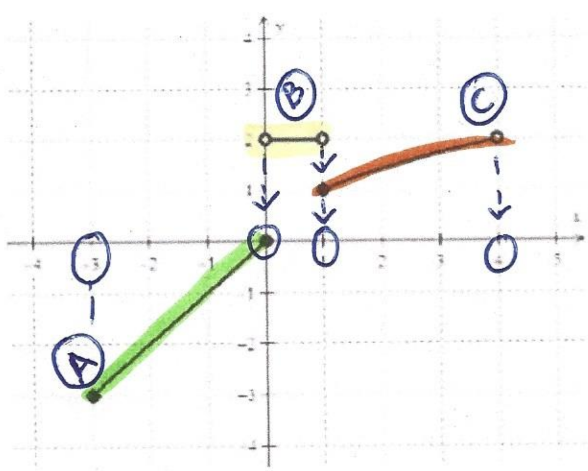
$x < 4$

domain: $(-\infty, 4)$

range: $[-2, \infty)$

$y \geq -2$

3. Define the piecewise function below.



$$f(x) = \begin{cases} x \,; & -3 \le x \le 0 \\ 2 \,; & 0 < x < 1 \\ \sqrt{x} \,; & 1 \le x < 4 \end{cases}$$

4. What value of ***a*** would make this piecewise function *continuous*?

$$f(x) = \begin{cases} 3x^2 + 4 & \text{if } x < -2 \\ 5x + a & \text{if } x \geq -2 \end{cases}$$

$f(x) = 3x^2 + 4$
$= 3(-2)^2 + 4$
$= 16$

$(-2, 16)$

use $(-2,16)$ to find a

$f(x) = 5x + a$
$16 = 5(-2) + a$
$16 = -10 + a$
$a = 26$

5. Your dog groomer charges according to your dog's weight. If your dog is 15 pounds and under, the groomer charges $35. If your dog is between 15 and 40 pounds, she charges $40. If your dog is over 40 pounds, she charges $40 plus an additional $2 for each pound over 40.

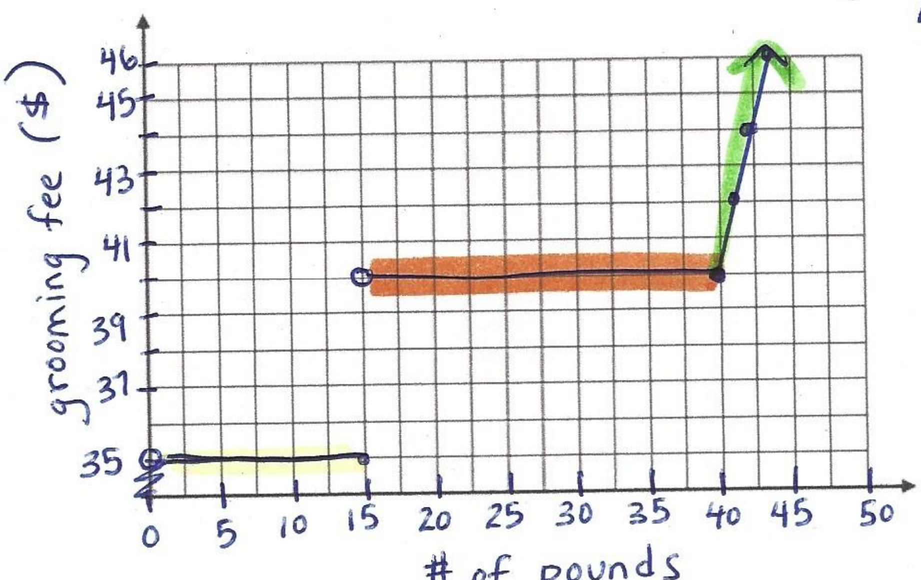
a. Write the piecewise function that defines the amount your groomer charges.

$$f(x) = \begin{cases} 35 \;; & x \leq 15 \\ 40 \;; & 15 < x \leq 40 \\ 2(x-40) + 40 \;; & x > 40 \end{cases}$$

simplified → $2x - 40$

b. Graph the piecewise function

Dog Grooming Fee
According to weight



$f(x) = 2x - 40$

X	Y
40	40
41	42
42	44
43	46

c. How much would the groomer charge if your dog weighs 60 pounds?

use $f(x) = 2x - 40$

$x = 60$

$f(x) = 2(60) - 40$
$= 80$

You pay $80 to have a 60 pound dog groomed