Algebra RH

Essential Question: What is the discriminant?

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Do Now: Using the quadratic formula, find the solution(s) to the following equations

a. $x^2 - 3x - 4 = 0$

$a = 1$, $b = -3$, $c = -4$

$x = \dfrac{3 \pm \sqrt{(-3)^2-4(1)(-4)}}{2(1)}$

$x = \dfrac{3 \pm \sqrt{25}}{2}$

$x = \dfrac{3 \pm 5}{2}$

$\dfrac{3+5}{2}$ $\quad$ $\dfrac{3-5}{2}$

$x = 4$ or $x = -1$

b. $-x^2 + 2x - 1 = 0$

$a = -1$, $b = 2$, $c = -1$

$x = \dfrac{-2 \pm \sqrt{(2)^2-4(-1)(-1)}}{2(-1)}$

$x = \dfrac{-2 \pm \sqrt{0}}{-2}$

$x = \dfrac{-2}{-2}$

$x = 1$

c. $2x^2 - 2x + 3 = 0$

$a = 2$, $b = -2$, $c = 3$

$x = \dfrac{2 \pm \sqrt{(-2)^2-4(2)(3)}}{2(2)}$

$x = \dfrac{2 \pm \sqrt{-20}}{4}$

$x = \dfrac{2 \pm 2i\sqrt{5}}{4}$

$x = \dfrac{1 \pm i\sqrt{5}}{2}$

How many solutions can a quadratic equation have?

Before solving a quadratic equation, the *discriminant* can be used to determine the number of real solutions.

The *discriminant* is the expression under the radical in the quadratic formula.

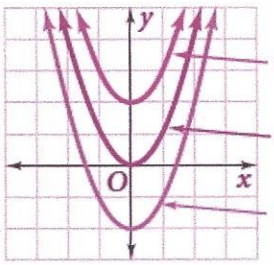
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ← the discriminant b^2-4ac

To determine the number of solutions:

If $b^2 - 4ac > 0$, there are ___two___ solutions.

If $b^2 - 4ac = 0$, there is ___one___ solution.

If $b^2 - 4ac < 0$, there are ___no real___ solutions.
(two imaginary)



Discriminant is negative.

Discriminant is 0.

Discriminant is positive.

To determine the type of solutions:

If $b^2 - 4ac$ is a perfect square, the solutions are ___rational___ numbers.

If $b^2 - 4ac$ is not a perfect square, the solutions are ___irrational___ numbers.

For each quadratic equation below:

a) Find the value of the discriminant and determine how many solutions the equation has.

b) Determine if the solution(s) are rational or irrational.

1. $x^2 - 2x + 4 = 0$

$a = 1$, $b = -2$, $c = 4$

$b^2 - 4ac$

$(-2)^2 - 4(1)(4)$

$4 - 16$

-12 ← negative

(two) imaginary solutions

[imaginary]

2. $-3x^2 + 5x - 1 = 0$

$a = -3$, $b = 5$, $c = -1$

$b^2 - 4ac$

$(5)^2 - 4(-3)(-1)$

$25 - 12$

13 ← positive non perfect square

two irrational solutions

[real, irrational, unequal] (2 distinct answers)

3. $-x^2 - 10x - 25 = 0$

$a = -1$, $b = -10$, $c = -25$

$b^2 - 4ac$

$(-10)^2 - 4(-1)(-25)$

$100 - 100$

0 ← zero

one solution

[real, rational, equal] (one distinct answer)

4. $2x^2 + 10x = -12$

$2x^2 + 10x + 12 = 0$

$a = 2$, $b = 10$, $c = 12$

$b^2 - 4ac$

$(10)^2 - 4(2)(12)$

$100 - 96$

4 ← positive perfect square

two rational solutions

[real, rational, unequal]

5. $3x^2 - 2x = -5$

$3x^2 - 2x + 5 = 0$

$a = 3$, $b = -2$, $c = 5$

$b^2 - 4ac$

$(-2)^2 - 4(3)(5)$

$4 - 60$

-56 ← negative

two imaginary solutions

[imaginary]

6. $x^2 + 14 = -9x$

$x^2 + 9x + 14 = 0$

$a = 1$, $b = 9$, $c = 14$

$b^2 - 4ac$

$(9)^2 - 4(1)(14)$

$81 - 56$

25 ← positive perfect square

two rational solutions

[real, rational, unequal]