Algebra RH

Essential Question: How do we solve quadratic equations by completing the square?

Do Now: Solve for x.

1. $x^2 - 5x - 14 = 0$

$(x-7)(x+2)=0$

$x-7=0 \quad x+2=0$

$\boxed{x=7 \text{ or } x=-2}$

2. $2(x+1)^2 = 18$

$\sqrt{(x+1)^2}=\sqrt{9}$

$x+1 = \pm 3$

$x=-1\pm 3$

$\boxed{x=2 \text{ or } x=-4}$

3. $x^2 + 8x + 16 = 0$

$(x+4)(x+4)=0$

$x+4=0 \quad x+4=0$

$x=-4 \text{ or } x=-4$

"double solution"

Solving Quadratic Equations by Completing the Square

1. Refer to #3 from the Do Now. Is the quadratic trinomial $x^2 + 8x + 16$ a perfect square?

i.e. 25 is a perfect square because $5^2=25$

$(x+4)^2$ ← yes

$(x+4)(x+4)$

2. What value for c would make each quadratic trinomial a perfect square?

a. $x^2 + 6x + \underline{9}$ ($\frac{1}{2}$ of $6 = 3$, $3^2=9$)

$(x+3)^2$

b. $x^2 - 10x + \underline{25}$ ($\frac{1}{2}$ of $10 = 5$, $5^2=25$)

$(x-5)^2$

c. $x^2 - 1x + \underline{\frac{1}{4}}$ ($\frac{1}{2}$ of $1 = \frac{1}{2}$, $(\frac{1}{2})^2 = \frac{1}{4}$)

$(x-\frac{1}{2})^2$

How do we solve quadratic equations by completing the square?

$ax^2 + bx + c = 0$ (must be 1)

is more difficult when b is odd

1) Write the original equation (make sure the constant is on the right side)

$x^2 - 5x - 14 = 0$

$\rightarrow x^2 - 5x = 14$

2) Create a perfect square on the left hand side of the equation (take half the middle term and square it). Make sure to add the same amount to the right hand side of the equation.

$x^2 - 5x + \frac{25}{4} = 14 + \frac{25}{4}$

$(\frac{5}{2})^2$

"Half it, Square it, Share it"

3) Factor the left hand side of the equation.

$(x-\frac{5}{2})^2 = \frac{81}{4}$

4) Find the square root of each side.

$x - \frac{5}{2} = \pm\frac{9}{2}$

5) Solve for x.

Same as do now #1

$x = \frac{5}{2} \pm \frac{9}{2}$

$x = \frac{14}{2} \text{ or } x = \frac{-4}{2}$

$x=7 \text{ or } x=-2$

$$x^2 + 4x - 21 = 0$$

Factoring

$(x+7)(x-3)=0$

$x+7=0 \quad x-3=0$

$x=-7$ or $x=3$

Quadratic Formula

$a=1$

$b=4$

$c=-21$

$$x = \frac{-4 \pm \sqrt{4^2-4(1)(-21)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{100}}{2}$$

$$x = \frac{-4 \pm 10}{2}$$

$x=3$ or $x=-7$

Complete the square

$x^2+4x = 21$

$x^2+4x+\_\_\_ = 21+\_\_\_$

$\left(\frac{4}{2}\right)^2=4$

$x^2+4x+4 = 21+4$

$(x+2)^2 = 25$

$x+2 = \pm 5$

$x = -2 \pm 5$

$x=-7$ or $x=3$

1) $x^2 - 6x - 8 = 0$

$x^2 - 6x = 8$

$x^2 - 6x + \_\_ = 8 + \_\_$

$x^2 - 6x + 9 = 8 + 9$

$(x-3)^2 = 17$

$x - 3 = \pm\sqrt{17}$

$x = 3 \pm \sqrt{17}$

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2) $\frac{3x^2}{3} + \frac{6x}{3} - \frac{21}{3} = \frac{0}{3}$

$a = 3$ but needs to be 1

$x^2 + 2x - 7 = 0$

$x^2 + 2x = 7$

$x^2 + 2x + \_\_ = 7 + \_\_$

$x^2 + 2x + 1 = 7 + 1$

$(x+1)^2 = 8$

$x + 1 = \pm\sqrt{8}$

$x = -1 \pm \sqrt{8}$

$\boxed{x = -1 \pm 2\sqrt{2}}$

Example #2: $x^2 + 3x - 4 = 0$

$x^2 + 3x = 4$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ $\quad x^2 + 3x + \frac{9}{4} = 4 + \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{25}{4}$

$x + \frac{3}{2} = \pm\frac{5}{2}$

$x = -\frac{3}{2} \pm \frac{5}{2}$

$x = \frac{-3+5}{2}$ or $x = \frac{-3-5}{2}$

$\boxed{x = 1 \text{ or } x = -4}$ ← since solutions are rational, the quadratic must have been factorable

$(x+4)(x-1) = 0$

$x+4=0 \quad x-1=0$

$x=-4 \quad x=1$

↑ much easier

If factorable, just factor

Example #3: $\frac{2x^2}{2} - \frac{11x}{2} + \frac{12}{2} = \frac{0}{2}$

$x^2 - \frac{11}{2}x + 6 = 0$

$x^2 - \frac{11}{2}x + \frac{121}{16} = -6 + \frac{121}{16}$

$\left(\frac{11}{2} \cdot \frac{1}{2}\right)^2$ $\quad \left(x - \frac{11}{4}\right)^2 = \frac{25}{16}$

$\left(\frac{11}{4}\right)^2 = \frac{121}{16}$

$x - \frac{11}{4} = \pm\frac{5}{4}$

$x = \frac{11}{4} \pm \frac{5}{4}$

$x = \frac{16}{4}$ or $x = \frac{6}{4}$

$\boxed{x = 4 \text{ or } x = 3/2}$

Note: If the leading coefficient of $ax^2 + bx + c = 0$ is ***not*** 1, divide each side of the equation by the leading coefficient before completing the square!

$Ac = 24$

$2x^2 - 8x - 3x + 12 = 0$

$2x(x-4) - 3(x-4) = 0$

$(2x-3)(x-4) = 0$

$2x-3=0 \quad x-4=0$

$2x = 3$

$\boxed{x = \frac{3}{2} \text{ or } x = 4}$

Same