Algebra RH

Essential Question: How do we factor by grouping?

Do Now: Can you factor the following polynomial expressions?

A. $x^2(x+3)+2(x+3)$

$(x^2+2)(x+3)$

B. $3x^4+3x^3-7x-7$

C. $3x^2+10x+8$

$(3x+8)(3x+1)$
$(3x+1)(3x+8)$
$(3x+4)(3x+2)$
$(3x+2)(3x+4)$

Think about this...
Can we factor the expression $x^2(x+3)+2(x+3)$?

(a) Are there any common factors in the expression? $x+3$
Hint: Sometimes the greatest common factor is not a monomial.

(b) Rewrite the above expression as the product of two binomials.

$(\underbrace{x+3}_{\text{GCF}})(\underbrace{x^2+2}_{\text{What's left?}})$

➢ Let's practice factoring polynomials where the GCF is ***not*** a monomial.

Rewrite each of the following expressions as the product of two binomials by factoring out a ***common binomial factor***.

1) $2x(x-5)+3(x-5)$

$(x-5)(2x+3)$

2) $x^3(x-9)+(x-9)$

$(x^3+1)(x-9)$

3) $(x+5)(x+1)+(x+5)(x+8)$

$(x+1+x+8)(x+5)$

$(2x+9)(x+5)$

In some cases the GCF for ALL the terms of a polynomial is 1. If you have a four term polynomial and the GCF = 1, then try factoring by grouping.

Factoring a Polynomial with Four Terms by Grouping

Step 1: Group the first two terms together and then the last two terms together.

Step 2: Factor out a GCF from each separate binomial.

Step 3: Factor out the common binomial.

Example: Factor $3x^4 + 3x^3 - 7x - 7$

- $3x^4 + 3x^3 - 7x - 7$ ← group the first two and last two terms
- $3x^3(x + 1) - 7(x + 1)$ ← then factor out the GCF from each pair of terms
- $(x + 1)(3x^3 - 7)$ ← factor out the matching factor and write what is left (include parentheses around each factor).

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4) $x^3 - 2x^2 - 3x + 6$

$x^2(x-2) \mid -3(x-2)$

$(x-2)(x^2-3)$

5) $2x^3 - x^2 - 6x + 3$

$x^2(2x-1) \mid -3(2x-1)$

$(2x-1)(x^2-3)$

6) $5x^3 - 10x^2 + 3x - 6$

$5x^2(x-2) \mid +3(x-2)$

$(x-2)(5x^2+3)$

Factoring a Polynomial with Three Terms by Grouping	
Factoring a trinomial of the form $ax^2 + bx + c$, $a \neq 1$	**Factor: $3x^2 + 10x + 8$**
1. Always begin by factoring out the GCF	*The GCF of this polynomial is 1*
2. Find the product of a and c $(a \cdot c)$	$a \cdot c$ $3 \cdot 8 = 24$
3. Find two factors of ac that add up to b Find two factors of **+24** that sum to **+10**	$6 \cdot 4 = 24$ $6 + 4 = 10$
4. Replace the middle term with an equivalent expression that uses the integer pair found in the previous step	$3x^2 + 6x + 4x + 8$ *the order of middle terms does not matter*
5. Group the four terms to form two pairs	$3x^2 + 6x \quad + 4x + 8$
6. Factor each pair of terms by finding the GCF	$3x(x + 2) + 4(x + 2)$
7. Factor out the common (*shared*) binomial	$(x + 2)(3x + 4)$

7) $3x^2 + 14x - 5$ $\quad AC = -15$, $B = 14$

$3x^2 - x + 15x - 5$

$x(3x-1) \mid +5(3x-1)$

$(3x-1)(x+5)$

8) $2x^2 + 5x + 2$ $\quad AC = 4$, $B = 5$

$2x^2 + 4x + x + 2$

$2x(x+2) \mid +1(x+2)$

$(2x+1)(x+2)$

9) $16x^2 + 8x + 1$

AC = 16
B = 8

$16x^2 + 4x + 4x + 1$

$4x(4x+1) + 1(4x+1)$

$(4x+1)(4x+1)$

10) $4x^2 - 12x + 5$

AC = 20
B = -12

$4x^2 - 10x - 2x + 5$

$2x(2x-5) - 1(2x-5)$

$(2x-5)(2x-1)$

11) $2x^2 + x - 15$

AC = -30
B = 1

$2x^2 + 6x - 5x - 15$

$2x(x+3) - 5(x+3)$

$(2x-5)(x+3)$

11) $4x^2 - 5x - 6$

AC = -24
B = -5

$4x^2 - 8x + 3x - 6$

$4x(x-2) + 3(x-2)$

$(4x+3)(x-2)$

12) $3x^2 - 7x + 2$

AC = 6
B = -7

$3x^2 - 6x - x + 2$

$3x(x-2) - 1(x-2)$

$(3x-1)(x-2)$

13) $6x^2 - 17x + 12$

AC = 72
B = -17

$6x^2 - 9x - 8x + 12$

$3x(2x-3) - 4(2x-3)$

$(3x-4)(2x-3)$