Algebra RH

Essential Question: How can we factor polynomials?

**Do Now:** Find the GCF of each set of terms.

(a) $36a^4b^3$ and $48a^2$

$12a^2$

(b) $14x^2y^4$ and $21xy^3$

$7xy^3$

## Let's Review Some Important Vocabulary

**Factors:** Numbers and variables that when multiplied together produce a given product.

Examples: Factors of 36 = {1,2,3,4,6,9,12,18,36} Factors of $6x$ = {$1,6x,x,6,2x,3,2,3x$}

**Integral Factors:** Factors that are integers.

Example: The integral factors of 4 = {1, 2, 4, -1, -2, -4}

**Factoring:** Rewriting a polynomial expression as a product.

Example: $6x^2 + 3$ in factored form => $3(2x^2 + 1)$

**Prime Polynomials:** A polynomial is prime if it cannot be written as a product of polynomials with integer coefficients.

Example: $2x + 5$ Non-example: $2x + 4$ can be factored into $2(x + 2)$

## Factoring Polynomials by factoring out a Monomial (GCF) *GCF(__________)*

- **Determine the GCF of each term (1st factor)**
- **Divide each term by the GCF in order to find the second factor**

Examples:

1. $9a^4 + 6a^3 - 12a$

$3a(3a^3 + 2a^2 - 4)$

2. $4x^2y^3 - 2xy$

$2xy(2xy^2 - 1)$

3. $64x^3 - 56x^2 + 88x$

$8x(8x^2 - 7x + 11)$

4. $18abc + 4bc - 2a^2bc^2$

$2bc(9a + 2 - a^2c)$

5. $24x^3 + 32x + 15$

prime polynomial

6. $a^3b^2 + a^3b^4 + ab^4$

$ab^2(a^2 + a^2b^2 + b^2)$

## Factoring a Quadratic Trinomial (coefficient of $x^2$ is 1) $x^2 + bx + c$

**AM Method**

$\mathbf{x^2 + bx + c}$ factors into 2 binomials $\mathbf{(x + p)(x + q)}$ where $\mathbf{p + q = b}$ and $\mathbf{pq = c}$

Examples:

7. $x^2 + 11x + 28$

$(x+7)(x+4)$

| 28 | |
|---|---|
| 1 | 28 |
| 2 | 14 |
| 4 | 7 |

8. $x^2 - 9x + 8$

$(x-8)(x-1)$

9. $x^2 + x - 20$

$(x+5)(x-4)$

10. $x^2 - 10x + 21$

$(x-7)(x-3)$

11. $x^2 + 14x + 40$

$(x+4)(x+10)$

12. $x^4 - 2x^2 - 15$

$(x^2-5)(x^2+3)$

13. $x^2 - 33x - 280$

$(x-40)(x+7)$

14. $x^2 + 8xy - 33y^2$

$(x+3y)(x+11y)$

## Factoring the Difference of Two Squares ("DOTS") $a^2 - b^2$

- **In order to factor DOTS, you much recognize DOTS.**

  Example: Is $x^2$ - 9 a difference of two squares (DOTS)?

  Both $x^2$ and 9 are perfect squares. Since we are subtracting the perfect squares, this expression is referred to as the **difference of two squares.**

- **Once you have DOTS, take the square root of each term.**

In $x^2$ - 9...
- What times itself is $x^2$? $x$
- What times itself is 9? 3

**List the perfect squares...**

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

$x^2, x^4, x^6, x^8, x^{10}$...
multiples of 2

- **Using each root, create a sum and difference.**

The factors are ___X___ and ___3___.

Therefore, the factorization of $x^2$ - 9 is ___$(x+3)(x-3)$___.

**Rule:** $a^2 - b^2$ = ___$(a-b)(a+b)$___

Factor the following expressions using the **DOTS** method.

| | |
|---|---|
| 15) $x^2 - 1$ <br> $(x-1)(x+1)$ | 16) $81x^2 - 25$ <br> $(9x-5)(9x+5)$ |
| 17) $64x^2 - 9$ <br> $(8x-3)(8x+3)$ | 18) $x^2 - y^4$ <br> $(x-y^2)(x+y^2)$ |
| 19) $x^2y^4 - 16$ <br> $(xy-4)(xy+4)$ | 20) $144 - a^2$ <br> $(12-a)(12+a)$ |
| 21) $25a^2 - 16b^2$ <br> $(5a-4b)(5a+4b)$ | 22) $49x^4 - 144y^8$ <br> $(7x^2-12y^4)(7x^2+12y^4)$ |

23) A rectangle has an area of $16x^2 - 64$. What could be the dimensions of the rectangle?

$(4x-8)(4x+8)$

24) Is $a^2 + b^2$ factorable? Explain.

No, not a difference of perfect squares

25) Is $x^9 - 4$ factorable? Explain.

No, $x^9$ is not a perfect square