

Algebra RH

Essential Question: How can we represent an exponential function that shows an increase or decrease over time?

Do Now: Read and solve the following word problems.

- (a) The radio station Z-100 is sponsoring a contest. The prize begins as a \$1000 gift card to Roosevelt Field Mall. Once a day, the disc jockey announces a name, and the person has 15 minutes to call in and claim the prize. If the person does not call within the allotted time, the prize increases by 10% per day. How much will the gift card be worth if no one wins after 3 days?

| Number of Days Passed | Mathematical Expression | Prize Money |
|-----------------------|--------------------------------------|-------------|
| 0 | <i>Initial Value of the Prize</i> | \$1000 |
| 1 | $1000 (100\% + 10\%) \leftarrow 1.1$ | 1100 |
| 2 | $1000 (1.1) (1.1)$ | 1,210 |
| 3 | $1000 (1.1) (1.1) (1.1)$ | 1,331 |

- (b) Leo purchases a car for \$26,499. The car depreciates (loses value) at a rate of 18% annually. What will Leo's car be worth after 3 years?

| Number of Years | Mathematical Expression | Car's Worth |
|-----------------|----------------------------|--------------|
| 0 | <i>Initial Value</i> | \$26,499 |
| 1 | $26,499 (.82)$ | \$ 21,729.18 |
| 2 | $26,499 (.82) \cdot (.82)$ | 17,817.93 |
| 3 | $26,499 (.82) (.82) (.82)$ | 14,610.70 |

losing 18% value → retains 82% value

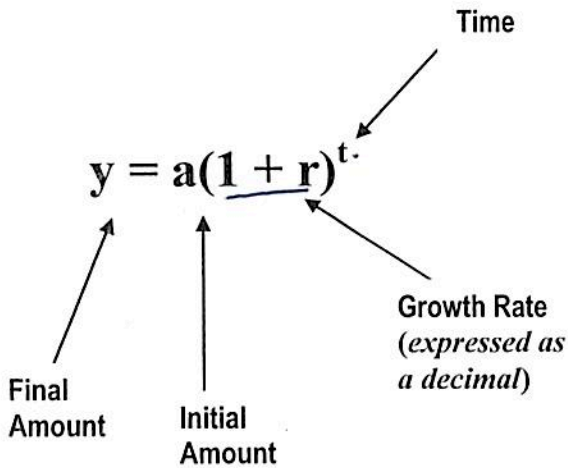


Exponential Growth occurs when a quantity increases by the same rate, r , in each unit of time, t . When this happens, the value of the quantity at any given time can be calculated as a function of the rate and the original amount.

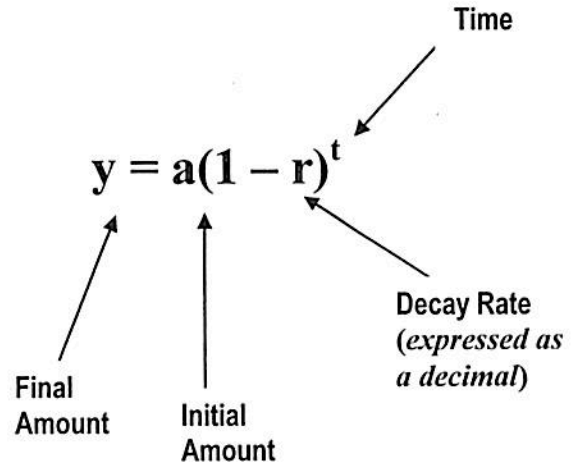
Exponential Decay occurs when a quantity decreases by the same rate, r , in each unit of time, t . When this happens, the value of the quantity at any given time can be calculated as a function of the rate and the original amount.

$$y = ab^x$$

Exponential Growth Model



Exponential Decay Model



Let's look at the *Do Now...* which situation represents exponential growth? exponential decay?

Exponential Model

$$y = a(1+r)^t$$

$$y = 1000(1+.10)^t$$

$$y = 1000(1.1)^t$$

What is the value of the prize money after 3 days have passed?

$t=3$

$$y = 1000(1.1)^3$$

$$= \$1331$$

Exponential Model

$$y = a(1-r)^t$$

$$y = 26,499(1-.18)^t$$

$$y = 26,499(.82)^t$$

What is the value of Leo's car after 3 years?

$t=3$

$$y = 26,499(.82)^3$$

$$= \$14,610.70$$

Examples:

- A sculpture was valued at \$1200 in the year 1990. Since then it has been appreciating at a rate of 8% per year.

- Write an exponential growth function to model this situation.

$a = 1200$

$r = .08$

initial value "a"

$$y = 1200(1.08)^t$$

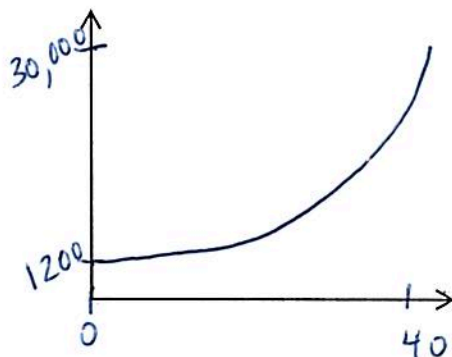
factor = 1.08

b) Complete the table of values that shows the increase in value over time.

| t (time in yrs) | 0 | 5 | 10 | 15 | 20 |
|-----------------|------|---------|---------|---------|---------|
| y (value in \$) | 1200 | 1763.19 | 2590.71 | 3806.60 | 5593.15 |

c) Sketch a graph of the function with the indicated window.

Window:
 x-min: 0
 x-max: 40
 xscl: 5
 y-min: 1,200
 y-max: 30,000
 yscl: 150



Think About This?

What do the window settings mean in the context of the problem?

d) How much is the sculpture worth now?

$$a = 1200$$

$$r = .08$$

$$t = 2020 - 1990 = 30$$

$$y = 1200(1.08)^{30}$$

$$y = 12,075.19$$

2. Mr. Rogers purchased machinery for his farming operation for \$175,000. It is expected to depreciate at a rate of 18% per year. Write an exponential function to model this situation. What will be the value of the machinery in 10 years?

$$a = 175,000$$

$$r = .18 \downarrow$$

$$\text{factor} = .82 \quad t = 10$$

$$y = 175,000(.82)^{10}$$

$$= \$24,053.41$$

3. Maria's parents invested \$14,000 in a CD account earning 6% per year compounded annually. How much money will there be in the account after 10 years?

$$a = 14,000$$

$$r = .06 \uparrow$$

$$\text{factor} = 1.06 \quad t = 10$$

$$y = 14,000(1.06)^{10}$$

$$y = 25,071.87$$

4. In 2000, 2200 students attended Polaris High School. The enrollment has since been declining 2% annually. If this trend continues, how many students will be enrolled in 2015?

$$a = 2200$$

$$r = .02 \downarrow$$

$$\text{decay factor} = 0.98$$

$$t = 2015 - 2000 = 15$$

$$y = 2200(0.98)^{15}$$

$$y = 1624.852026$$

$$\approx 1624 \text{ students}$$



If a relationship grows over time, it can be represented by an **Exponential**

Growth model, $y = a(1+r)^t$, where $1+r$ represents the growth factor between successive function values when t increases by 1.

If a relationship decreases over time, it can be represented by an **Exponential**

Decay model, $y = a(1-r)^t$, where $1-r$ represents the decay factor between successive function values when t increases by 1.

Algebra RH
Exponential Growth/Decay

HW # _____

For #1-2, identify if the situation represents Then state the for each.

| | | exponential growth or decay? | growth/decay factor? |
|----|--|------------------------------|--------------------------------------|
| 1. | Ms. Arnold received a job as a teacher with a starting salary of \$55,000. According to her contract, she will receive a 1.5% increase in her salary every year. | growth | $r = .015$ growth factor 1.015 |
| 2. | A fully inflated raft containing 4500 cubic inches of air loses 6.6% of its air every day. | | |

Read each word problem below. Use an exponential growth or decay model to solve.

- Camilo purchased a rare coin from a dealer for \$300. The value of the coin increases 5.5% each year. Determine the value of the coin in 5 years.
- In a particular state, the population of black bears has been decreasing at the rate of 0.75% per year. In 1990, it was estimated that there were 400 black bears in the state. If the population continues to decline at the same rate, what will the population be in 2017?