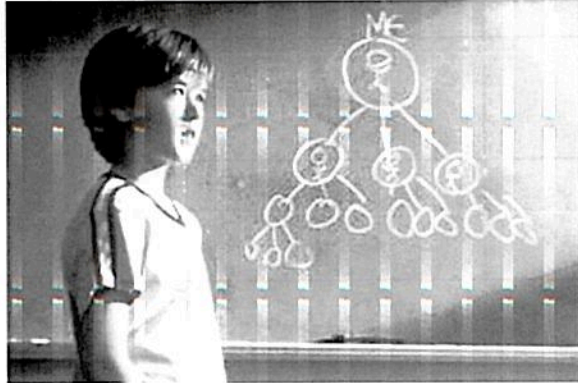


Essential Question: What is a geometric sequence?

Do Now: In the movie "Pay it Forward" the main character, a young boy, determines that he can make a significant difference in the world by creating a chain of events. During the movie he helps three people, who each help three people and so on.



(a) How many people's lives would be affected in the 6th round of this pattern?

1, 3, 9, 27, 81, 243

(b) Identify the pattern in this sequence of numbers.

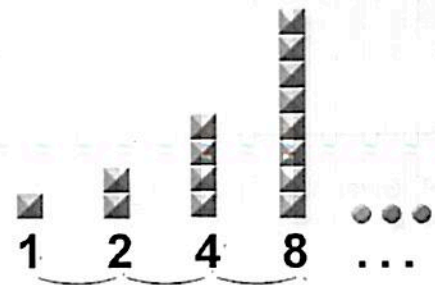
multiplying by 3

GEOMETRIC SEQUENCE

If a sequence of values follows a pattern of **multiplying** a fixed amount (not zero) times each term to arrive at the following term, it is referred to as a **geometric sequence**.

The numerical pattern that increases or decreases at a constant rate or value is called the **common ratio**.

To find the common ratio (r): Divide a term by the previous term.



The common ratio in this example is 2. The next term is 16.

Let's take a look at some sequences...is there a common ratio? If so, find the next two terms in the sequence.

(1) 1, -2, 4, -8, ...

yes, $r = -2$

16, -32

(2) 3, 6, 10, 15, ...

No,
no common
ratio

(3) 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...

yes, $r = \frac{1}{2}$

$\frac{1}{16}$, $\frac{1}{32}$

Finding Specific Terms of a Geometric Sequence

Complete the following chart for a common ratio of 3 and first term of 2.

| Term | Symbol | in terms of $a(1)$ and r | Substitution | Result |
|------|--------|--------------------------------|---|--------|
| 1 | $a(1)$ | $a(1)$ | - | 2 |
| 2 | $a(2)$ | $a(1) \cdot r^1$ | $a(1) \cdot r$ $2 \cdot 3$ 6 | 6 |
| 3 | $a(3)$ | $a(1) \cdot r \cdot r$ | $a(1) \cdot r^2$ $2 \cdot 3^2$ 18 | 18 |
| 4 | $a(4)$ | $a(1) \cdot r \cdot r \cdot r$ | $a(1) \cdot r^3$ $2 \cdot 3^3$ | 54 |
| : | : | : | : | : |
| n | $a(n)$ | $a(1) \cdot r^{n-1}$ | $a(1) \cdot r^{n-1}$ | |

Explicit Formula to find the n th term of a geometric sequence:

$$a(n) = a(1) \cdot r^{n-1}$$

(4) Given the following geometric sequence: 1, 4, 16, 64, ...

a) Define the sequence explicitly.

$$a(1) = \underline{1} \quad r = \underline{4}$$

$$a(n) = 1(4)^{n-1}$$

b) Find the 11th term. $n = \underline{11}$

$$\begin{aligned} a(11) &= 1(4)^{11-1} \\ &= 1(4)^{10} \\ &= 1,048,576 \end{aligned}$$

(5) Given the following geometric sequence: 128, 32, 8, 2, 0.5, ...

a) Define the sequence explicitly.

$$a(1) = \underline{128} \quad r = \underline{\frac{1}{4}}$$

$$a(n) = 128(.25)^{n-1}$$

b) Find the 8th term. $n = \underline{8}$

$$\begin{aligned} a(8) &= 128(.25)^{8-1} \\ &= 128(.25)^7 \\ &= 0.0078125 \end{aligned}$$

write the formula for this sequence that finds the value of any term

- (6) A ball is dropped from a height of 8 feet. The ball bounces to 50% of its previous height with each bounce. How high (to the nearest tenth of a foot) does the ball bounce on the fifth bounce?



$$a(1) = 4$$

$$r = \frac{1}{2} = 0.5$$

$$n = 5$$

$$a(n) = 4(0.5)^{n-1}$$

$$a(5) = 4(0.5)^{5-1}$$

$$= 4(0.5)^4$$

$$= 0.25$$

$$= 0.3$$

Partner Activity



You're sitting in Algebra class, minding your own business, when in walks a Bill Gates kind of guy - the real success story of Roslyn Middle School. He's made it big, and now he has a job offer for you!

He doesn't give too many details, but says that he is going to need you for 30 days, and you'll have to miss school. (Won't that be awful?)

You'll have your choice of two payment options:

> **Option 1:**

One cent on the 1st day, 2¢ on the 2nd day, 4¢ on the 3rd day and double your salary every day thereafter for 30 days.

or

> **Option 2:**

Exactly \$1,000,000 -- Yes, that's one million dollars!



You jump up out of your seat at that and say, "I'll take Option 1."

$$a(n) = .01(2)^{n-1}$$

$$a(30) = .01(2)^{30-1} \rightarrow .01(2)^{29} \rightarrow \$5,368,709.12$$

