

Algebra RH

Essential Questions: What is an arithmetic sequence? What is an explicit formula? How do we use explicit formulas to find the terms in a sequence?

Do Now:

During their routine, a high school marching band marches in rows. There is one performer in the first row, three performers in the second row, and five performers in the third row. This pattern continues for the rest of the rows.

(a) Write a recursive formula for this situation.

$$a_n = a_{n-1} + 2, a_1 = 1$$

(b) How many performers are in the 6th row?

$$a_3 = 5$$

$$a_4 = 5 + 2 = 7$$

$$a_5 = 7 + 2 = 9$$

$$a_6 = 9 + 2 = 11$$

11 performers

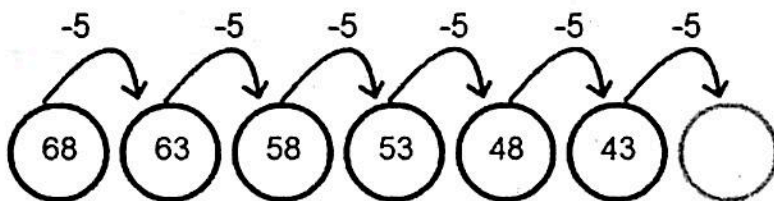
1, 3, 5, ...



What is an arithmetic sequence?

An **arithmetic sequence** is a numerical pattern that increases or decreases at a constant rate or value called the **common difference (d)**.

To find the common difference: Take one term and subtract the previous term.



The common difference in this example is -5.

The next term is 38.

Let's take a look at some sequences...is there a common difference?

(1) -2, -5, -8, -11, ...

yes

$$d = -3$$

(2) 16, 12, 9, 8, 4, 2, ...

No,
various
differences

(3) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}$

yes,

$$d = \frac{1}{4}$$

5, 7, 9, 11, 13...

Finding specific terms of an arithmetic sequence

Complete the following chart for a common difference of 2 and first term of 5.

Term #	Symbol	in terms of a(1) and d	Substitution	Result
1	a(1)	a(1)	-	5
2	a(2)	a(1) + d	a(1) + d 5 + 2 7	7
3	a(3)	a(1) + d + d	a(1) + 2d 5 + 2(2)	9
4	a(4)	a(1) + d + d + d	a(1) + 3d 5 + 3(2)	11
:	:	:	:	:
n	a(n)	a(1) + d(n-1)	a(n) = 5 + 2(n-1)	

Explicit Formula to find the *n*th term of an arithmetic sequence:

$$a(n) = a(1) + d(n-1)$$

$$a_n = a_1 + d(n-1)$$

(4) Given the following arithmetic sequence: 2, 6, 10, 14,...

a) Define the sequence explicitly.

a(1) = 2 d = 4

$$a(n) = 2 + 4(n-1)$$

write a formula that will find any term of this sequence

b) Find the 8th term. n = 8

$$a(8) = 2 + 4(8-1)$$

$$= 30$$

(5) Given the following arithmetic sequence: 100, 120, 140, 160,...

a) Define the sequence explicitly.

a(1) = 100 d = 20

~~a(n)~~
a_n = 100 + 20(n-1)

b) Find the 10th term.

$$a_{10} = 100 + 20(10-1)$$

$$= 280$$

$$a(1) = 15$$

$$d = 3$$

(6) The first row of the theater has 15 seats in it. Each subsequent row has 3 more seats than the previous row.

(a) Find the number of seats in the fifth row. $n=5$

$$a(n) = 15 + 3(n-1)$$

$$a(5) = 15 + 3(5-1) \\ = 27$$

(b) If the last row has 78 seats, how many rows are in the theater?

$$a(n) = 15 + 3(n-1)$$

$$78 = 15 + 3(n-1)$$

$$\cancel{78} = \cancel{15} \quad 63 = 3(n-1) \quad 21 = n-1 \\ n = 22$$

(7) $3+x, 9+3x, 13+4x, \dots$ is an arithmetic sequence for some real number x .

(a) Find the value of x .

$$9 + 3x - (3 + x)$$

$$9 + 3x - 3 - x$$

$$13 + 4x - (9 + 3x)$$

$$13 + 4x - 9 - 3x$$

$$\boxed{6 + 2x} = \boxed{4 + x}$$

$$6 + 2x = 4 + x$$

$$x = -2$$

(b) Find the 10th term.

$$a_1 = 1 \\ d = 2$$

$$a_n = 1 + 2(n-1)$$

$$a(n) = 1 + 2(n-1)$$

$$a(10) = 1 + 2(10-1) \\ = 19$$

The **TAKEAWAY**

- The increase or decrease in an arithmetic sequence is called the common difference.
- The explicit formula for an arithmetic sequence allows you to find the n th term of the sequence by substituting $a(1)$ and d in the expression.

↗
first term

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER!

1. If $f(x) = kx^2$, and $f(2) = 12$, then k equals A) 1 B) 2 C) 3 D) 4
2. If $f(x) = 3x + 4$, find $f(-4)$.
3. If $f(x) = 3x + 2$ and $g(x) = x - 3$, evaluate $f(g(x))$.
4. Using the functions $f(x) = 3x$ and $g(x) = x - 4$, demonstrate that the composition of these functions is not commutative.
5. Given the function rule $f(x) = x + 9$, find the range corresponding to the domain, $\{-3, 4, 6, 8\}$.
6. A caricaturist sets up an easel at a craft fair and quickly sketches portraits. The function below determines the amount of money, Q , she will receive at the end of the day after sketching m people. Given the function $Q(m) = 8m - 15$:
 - (a) Find $Q(3)$ and explain its meaning.
 - (b) Find m when $Q(m) = 41$ and explain its meaning.
7. A company produces tote bags. The fixed costs for producing the bags are \$12,000 and the variable costs are \$3 per tote bag.
 - (a) Write a function that describes the total cost, C , of producing b bags.
 - (b) Find the cost of producing 625 tote bags.
 - (c) Find how many tote bags can be produced with a budget of \$14,223.
8. A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which function(s) shown below can be used to determine the height, $f(n)$, of the sunflower in n weeks?
 - I. $f(n) = 2n + 3$
 - II. $f(n) = 2n + 3(n - 1)$
 - III. $f(n) = f(n - 1) + 2$ where $f(0) = 3$
 - a) I and II
 - b) II, only
 - c) III, only
 - d) I and III
9. The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_n , write an equation to find the m th term of this sequence.
10. Raymond is filling his kitchen sink to wash dishes. After one minute, there are 2.75 gallons of water in the sink. After two minutes, there are 5.5 gallons of water in the sink. After three minutes, there are 8.25 gallons of water in the sink. If this pattern continues, how many gallons of water will be in the sink after five minutes?